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A DIGITAL-COMPUTER MODEL FOR ESTIMATING HYDROLOGIC CHANGES IN THE AQUIFER SYSTEM IN DANE COUNTY, WISCONSIN

by

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U.S. GEOLOGICAL SURVEY

PREPARED BY
UNITED STATES DEPARTMENT OF THE INTERIOR
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WISCONSIN GEOLOGICAL AND NATURAL HISTORY SURVEY

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CONVERSION FACTORS

The following factors may be used to convert the English units published herein to the International System of Units (SI):

Multiply English units	By	To obtain SI units
million gallons per day (Mgal/d)	0.04381	cubic metres per second (m^3/s)
square miles (mi^2)	2.590	square kilometres (km^2)
feet (ft)	.3048	metres (m)
cubic feet per second (ft^3/s)	.02832	cubic metres per second (m^3/s)
gallons per day per foot [(gal/d)/ft]	.01242	metres squared per day (m^2/d)
gallons per day per square foot [$(gal/d)/ft^2$]	.04075	metres per day (m/d)

ABSTRACT

The extensive use of ground water for water supply within Dane County has resulted in the need for an appraisal of the area's ground-water resources. Water-resources planners and other water-oriented groups have expressed concern over ground-water level declines and reductions in streamflow that are occurring as a result of heavy pumping.

Digital-computer modeling techniques were used to estimate hydrologic changes in the aquifer system that would be caused by continued development. The system was modeled as a two-aquifer system consisting of a confined sandstone aquifer overlain by a leaky unconfined aquifer and underlain by impermeable bedrock. The physical properties of the aquifer system needed for the model were approximated using aquifer-test data and well-log data and by matching observed hydrologic changes in the system with corresponding changes computed by the model.

Computed hydrologic changes do not represent a serious depletion of the available ground-water supply for the foreseeable future. Maximum added regional declines in ground-water levels (drawdowns) from 1970 to 1990 were computed to be approximately 10 feet (3 metres) in the unconfined aquifer and approximately 40 feet (12 metres) in the confined aquifer. It is computed that for the same period the average annual streamflow from the upper Yahara River basin would be reduced by approximately 29 cubic feet per second (0.82 cubic metre per second). These changes are computed based on estimated development trends for the confined sandstone aquifer.

INTRODUCTION

Measurable hydrologic changes have occurred within Dane County as a result of pumping water from the aquifer system. Changes relative to predevelopment conditions have included reductions in regional ground-water levels in both the sandstone aquifer and overlying upper aquifer and reductions in base flow to streams.

Most changes have occurred within the upper Yahara River basin (fig. 1). Regional declines in water levels by 1970 in the sandstone aquifer ranged from less than 10 ft (3 m) to more than 70 ft (21 m). Declines in the overlying upper aquifer by the same year generally were less than 10 ft (3 m). Also by 1970, average streamflow of the Yahara River at the stream-gaging station was reduced about 34 percent because of ground-water pumping.

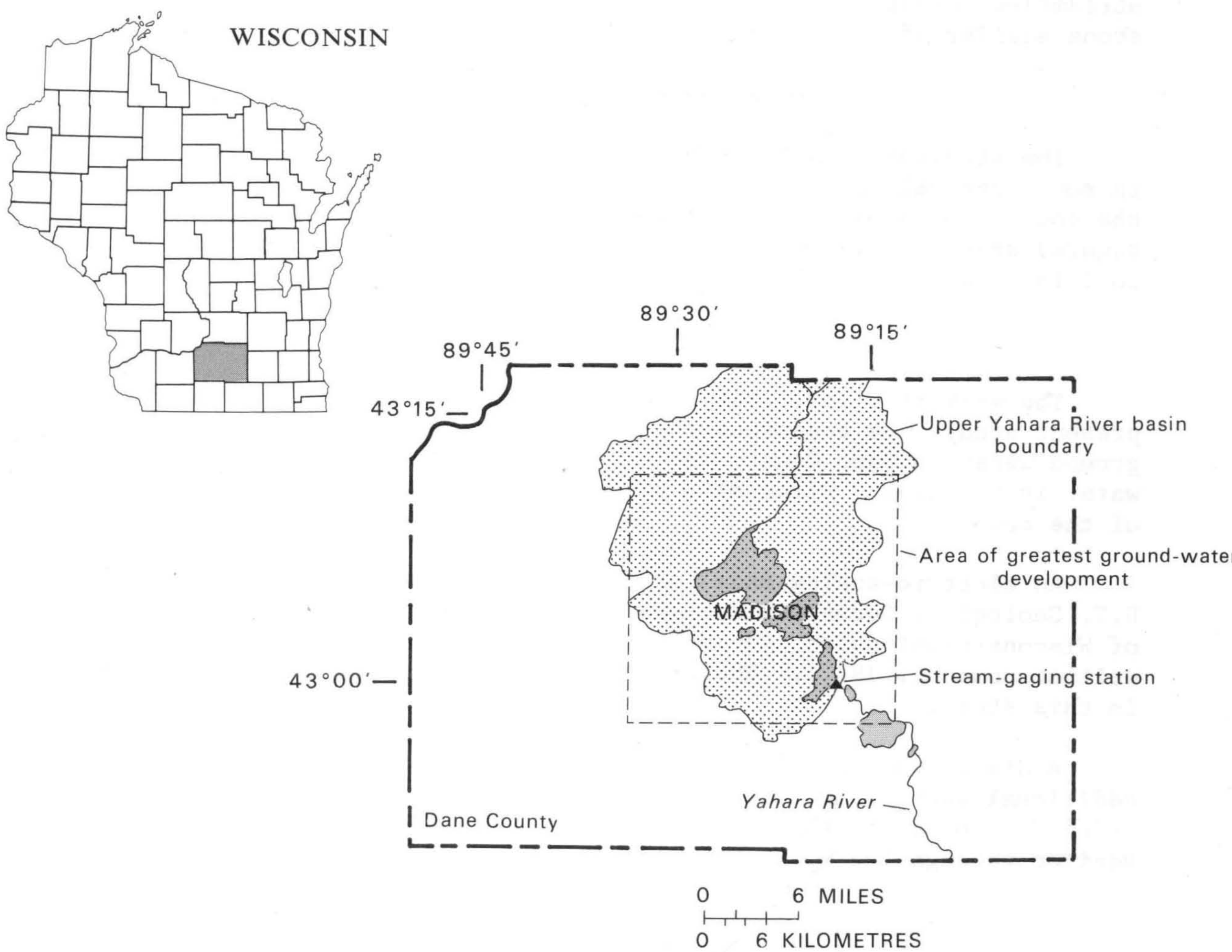


Figure 1. Location and extent of study area.

Purpose and Scope

The purpose of this report is to analyze the aquifer system in Dane County using digital-modeling techniques and to determine the probable future effects that continued pumping will have on the system.

The report describes development of the aquifer-system model through the use of algebraic finite-difference equations to estimate head changes in a coupled two-aquifer system. The upper and sandstone aquifer are coupled together using a quasi three-dimensional approach to the solution of the ground-water flow equations. The coupled aquifer model provides a more reliable representation of the aquifer system than modeling each aquifer separately. The equations are solved using a mathematical scheme similar to the one described by Bredehoeft and Pinder (1970).

The report also describes how the model was applied to the aquifer system to predict head declines in the upper and sandstone aquifers and reductions in streamflow through the year 1990 under proposed development plans for the sandstone aquifer of Dane County.

Location and Extent of Study Area

The study area is Dane County, an area of 1,233 mi^2 (3,192 km^2), located in south-central Wisconsin (fig. 1). The aquifer system was modeled throughout the county, with particular attention to a 16- X 17-mi square (26- X 27-km square) area roughly centered on Madison. Ground-water pumping is concentrated in this area.

Previous Investigations

The work of Cline (1965) provides good background information for the present study. He described the occurrence, movement, and availability of ground water in Dane County and the relationship between ground water and surface water in the area. Cline also presented a general description of the geology of the county.

An electric-analog model study of the Madison area was conducted by the U.S. Geological Survey in cooperation with the city of Madison and the University of Wisconsin-Extension, Geological and Natural History Survey (J. B. Gonthier, written commun., 1971). The sandstone aquifer in the Madison area was modeled in this study.

A digital-model study of the sandstone aquifer by McLeod (1975) provides additional background information for the present study. The digital-computer model developed for that study was used to predict drawdowns in the sandstone aquifer through the year 1990.

A DIGITAL-COMPUTER PROGRAM FOR SOLVING QUASI THREE-DIMENSIONAL GROUND-WATER FLOW PROBLEMS

The finite-difference technique is used to calculate approximate solutions to the partial-differential equations that describe head changes in a two-layer aquifer system that are caused by pumping. This technique involves artificially subdividing the aquifer system into a set of finite elements by superimposing a grid network (fig. 2) on the study area and then describing head changes in the two layers at each element using finite-difference approximations. The large number of resulting equations are solved using a digital computer.

Method of Analysis

The aquifer system to be analyzed consists of a confined sandstone aquifer overlain by an unconfined upper aquifer and underlain by impermeable bedrock (fig. 3). The potentiometric surface in the sandstone aquifer is different from the water table in the upper aquifer, and an exchange of water is occurring between the two aquifers.

Simplifying assumptions are made in order to develop two-dimensional flow equations for describing water movement in this aquifer system. These assumptions include:

1. Flow in both the upper and sandstone aquifers is horizontal even though vertical flow occurs. Also, areal variations in the water-table surface are small. These assumptions are justified because the horizontal extent of the aquifer system is much greater than the thickness of the system.

2. The exchange of water between the two aquifers occurs as vertical leakage. This assumption is reasonable in that the sandstone aquifer behaves in response to pumping as a confined aquifer that receives leakage from an overlying unconfined aquifer.

3. Water-table fluctuations have a negligible effect on ground-water evapotranspiration and infiltration from rainfall. This assumption is reasonable because the water table is located well below land surface throughout most of the area.

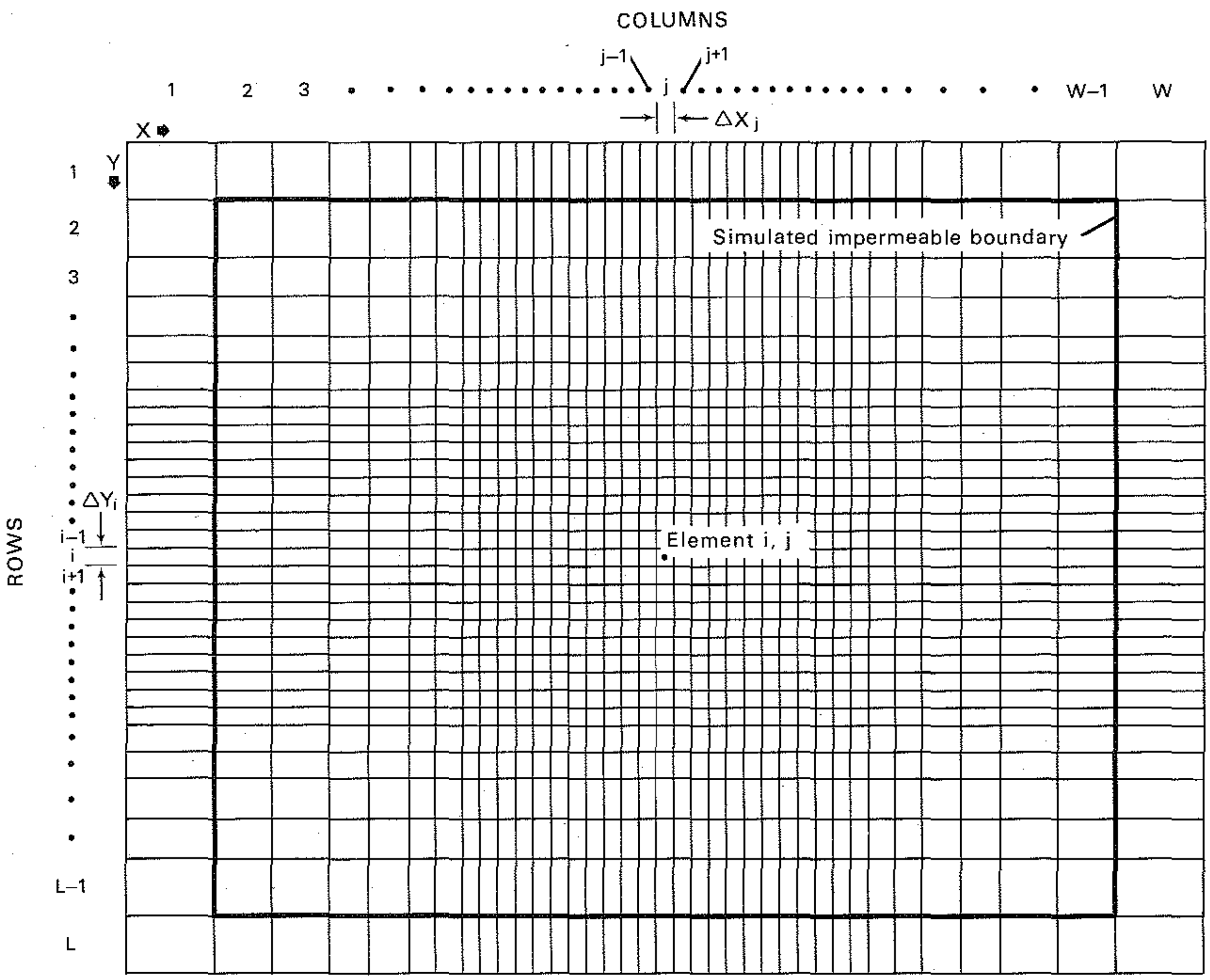
The partial-differential equation for describing head changes relative to predevelopment conditions in the sandstone aquifer caused by pumping using the above assumptions can be written as:

$$\frac{\partial}{\partial x}\left(T \frac{\partial s_c}{\partial x}\right) + \frac{\partial}{\partial y}\left(T \frac{\partial s_c}{\partial y}\right) = S \frac{\partial s_c}{\partial t} - W'\left(s_u - s_c\right) + Q_c \qquad (1)$$

where:

T = transmissivity for the sandstone aquifer defined as the rate at which water is transmitted through a unit width of the confined aquifer under a unit hydraulic gradient (Lohman and others, 1972, p. 13);

S = storage coefficient for the sandstone aquifer defined as the volume of water released from or taken into storage per unit surface area per unit change in head (Lohman and others, 1972, p. 13);



EXPLANATION

W=number of elements in a row

L=number of elements in a column

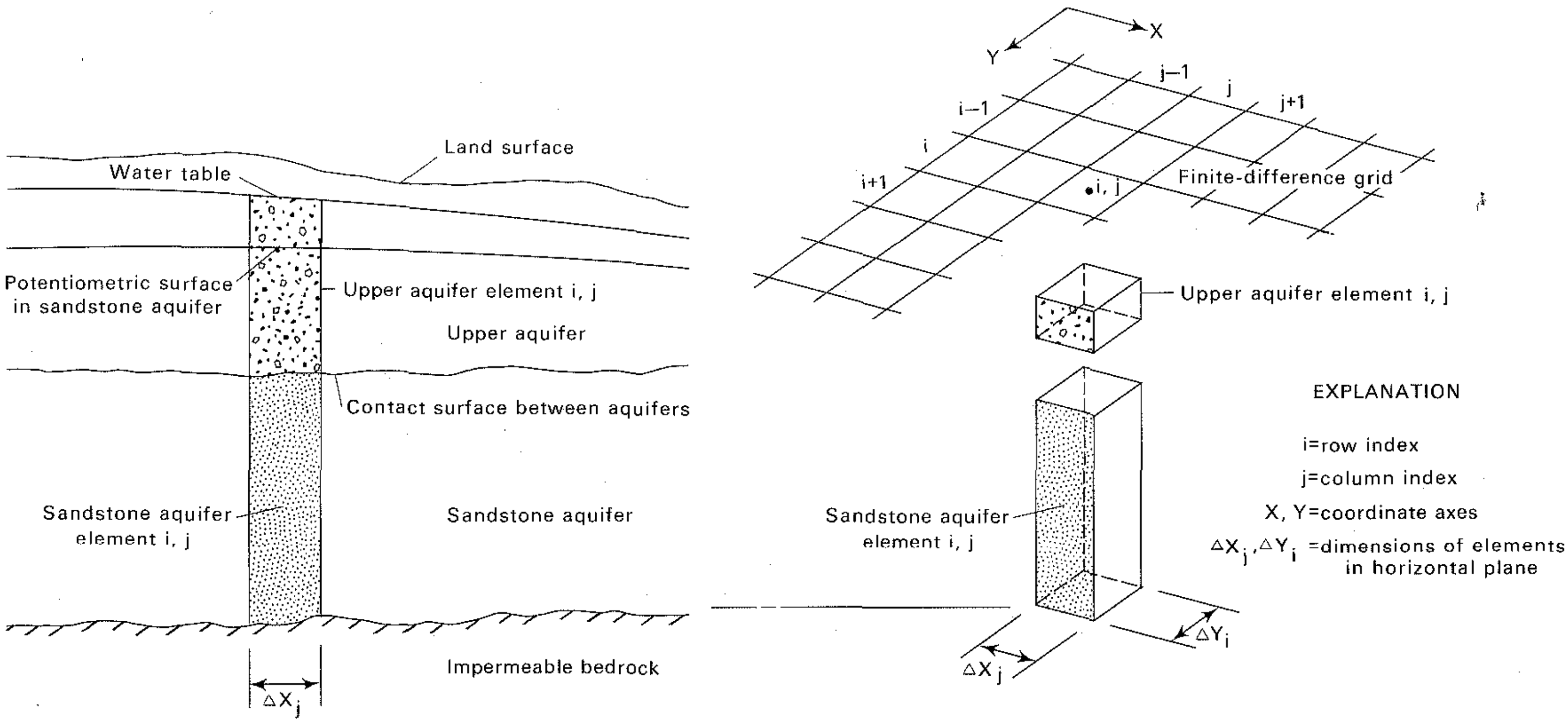
i=row index

j=column index

X,Y=coordinate axes

ΔX_j; ΔY_i =dimensions of elements in horizontal plane

Figure 2. Sample finite-difference grid.



A. Schematic representation of a continuous aquifer system

B. Finite element of the sandstone aquifer

Figure 3. Relation between a continuous aquifer system and a finite element of the system.

W' = leakage coefficient defined as the rate at which water flows through a unit horizontal area of contact surface between the upper aquifer and the sandstone aquifer when the difference between the head in the sandstone aquifer and the water table is unity (DeWiest, 1965, p. 274);

Q_c = pumping from sandstone aquifer;

x,y = horizontal reference axes;

s_c = head change in the sandstone aquifer caused by pumping;

s_u = head change in the upper aquifer caused by pumping; and

t = time since pumping began.

The equation for head changes relative to predevelopment conditions in the upper aquifer using these assumptions can be written as:

$$\frac{\partial}{\partial x}\left(Kb\ \frac{\partial s_u}{\partial x}\right) + \frac{\partial}{\partial y}\left(Kb\ \frac{\partial s_u}{\partial y}\right) = S_y\ \frac{\partial s_u}{\partial t} + W'\left(s_u - s_c\right) + Q_u \qquad (2)$$

where:

K = hydraulic conductivity of the upper aquifer defined as the rate at which water flows through a unit area, measured at right angles to the direction of flow, under a unit hydraulic gradient and at the prevailing kinematic viscosity (Lohman and others, 1972, p. 4);

b = saturated thickness of the upper aquifer;

S_y = specific yield of the upper aquifer defined as the ratio of the volume of water that the rock will yield to gravity drainage to the total volume of the rock (Lohman and others, 1972, p. 120); and

Q_u = pumping from the upper aquifer.

Other values in equation (2) are as previously defined.

A more general solution for water movement at any point in the aquifer system would require solving the flow equations in their three-dimensional form. The very large amount of computing time and computer-storage capacity required for a numerical solution of these equations (Bredehoeft and Pinder, 1970, p. 888), together with a lack of sufficient data to calibrate a three-dimensional model, precluded using such an approach.

Numerical Solution Process

The numerical solution process involves replacing each of the continuous variables in equations (1) and (2) with their discrete variable counterparts. The resulting finite-difference equations are solved numerically on a digital computer.

Equation (1) may be approximated by the finite-difference expression:

$$\frac{T'_{i,j-\frac{1}{2}}}{\Delta X_j}\left(s_{c_{i,j-1,k}} - s_{c_{i,j,k}}\right) + \frac{T'_{i,j+\frac{1}{2}}}{\Delta X_j}\left(s_{c_{i,j+1,k}} - s_{c_{i,j,k}}\right)$$

$$+ \frac{T'_{i-\frac{1}{2},j}}{\Delta Y_i}\left(s_{c_{i-1,j,k}} - s_{c_{i,j,k}}\right) + \frac{T'_{i+\frac{1}{2},j}}{\Delta Y_i}\left(s_{c_{i+1,j,k}} - s_{c_{i,j,k}}\right) \qquad (3)$$

$$= \frac{S_{i,j}}{\Delta t}\left(s_{c_{i,j,k}} - s_{c_{i,j,k-1}}\right) - W'_{i,j}\left(s_{u_{i,j,k}} - s_{c_{i,j,k}}\right) + \frac{Q_{c_{i,j}}}{\Delta X_j \Delta Y_i}$$

where:

i = row index;

j = column index;

k = time index;

$\Delta x; \Delta y$ = horizontal dimensions of aquifer elements, in units of length; and

$W'_{i,j}$ = leakage coefficient values for aquifer elements, in units of time to the minus one power.

$T'_{i+\frac{1}{2},j}$, for example, is the harmonic mean of $\frac{T_{i,j}}{\Delta Y_i}$; $\frac{T_{i+1,j}}{\Delta Y_{i+1}}$ and defined by:

$$T'_{i+\frac{1}{2},j} = \frac{2T_{i,j}T_{i+1,j}}{T_{i,j}\Delta Y_{i+1} + T_{i+1,j}\Delta Y_i}$$, in units of length per unit time;

$T_{i,j}$ = transmissivity values for elements of the sandstone aquifer, in units of length squared per unit time;

$S_{i,j}$ = storage coefficient values for elements of the sandstone aquifer, no dimensions;

$Q_{c_{i,j}}$ = pumping from sandstone aquifer elements, in units of length cubed per unit time. Recharge is negative, discharge is positive;

$s_{c_{i,j,k}}$ = head change in elements of the sandstone aquifer resulting from pumping, in units of length. Drawdowns are negative, rises are positive;

$s_{u_{i,j,k}}$ = head change in elements of the upper aquifer resulting from pumping, in units of length. Drawdowns are negative, rises are positive; and

Δt = time increment for calculation of head changes, in units of time.

Similarly, equation (2) may be expressed in finite-difference form as:

$$\frac{T''_{i,j-\frac{1}{2}}}{\Delta X_j}\left(s_{u_{i,j-1,k}} - s_{u_{i,j,k}}\right) + \frac{T''_{i,j+\frac{1}{2}}}{\Delta X_j}\left(s_{u_{i,j+1,k}} - s_{u_{i,j,k}}\right)$$

$$+ \frac{T''_{i-\frac{1}{2},j}}{\Delta Y_i}\left(s_{u_{i-1,j,k}} - s_{u_{i,j,k}}\right) + \frac{T''_{i+\frac{1}{2},j}}{\Delta Y_i}\left(s_{u_{i+1,j,k}} - s_{u_{i,j,k}}\right) \qquad (4)$$

$$= \frac{S_{y_{i,j}}}{\Delta t}\left(s_{u_{i,j,k}} - s_{u_{i,j,k-1}}\right) + W'_{i,j}\left(s_{u_{i,j,k}} - s_{c_{i,j,k}}\right) + \frac{Q_{u_{i,j}}}{\Delta X_j \Delta Y_i}$$

where:

$T''_{i+\frac{1}{2},j,k}$, for example, is the harmonic mean of $\frac{K_{i,j} b_{i,j,k}}{\Delta Y_i}$;

$\frac{K_{i+1,j} b_{i+1,j,k}}{\Delta Y_{i+1}}$ and defined as:

$$T''_{i+\frac{1}{2},j,k} = \frac{2 \quad K_{i,j} b_{i,j,k} \quad K_{i+1,j} b_{i+1,j,k}}{K_{i,j} b_{i,j,k} \Delta Y_{i+1} + K_{i+1,j} b_{i+1,j,k} \Delta Y_i}$$

, in units of length per unit time;

$K_{i,j}$ = hydraulic conductivity values for elements of the upper aquifer, in units of length per unit time;

$b_{i,j,k}$ = saturated thickness values for elements of the upper aquifer, in units of length;

$S_{y_{i,j}}$ = specific yield for upper aquifer elements, no dimensions; and

$Q_{u_{i,j}}$ = pumping from upper aquifer elements, in units of length cubed per unit time.

Other values in equation (4) are as previously defined.

Initial and Boundary Conditions

Initial head changes must be specified for every element in the finite-difference grid, and boundary conditions must be specified around the edges of the grid. Zero head changes are used as initial conditions because we are interested in changes relative to predevelopment conditions.

Impermeable boundary conditions are used around the edges of the grid. These boundary conditions are used to expedite the computational process. The physically appropriate boundary conditions are placed inside of this outer border.

Description of the Digital-Computer Program

The program begins by reading and printing information supplied on data cards. The data cards consist of a set of parameter cards and sets of aquifer data decks. The parameter cards contain data used to control computations and printouts from the model. The aquifer data decks contain data on the size of grid elements used in the model, physical properties of the aquifer system modeled, and pumping rates and locations.

The program then computes a set of iteration parameters based on the procedure outlined by Stone (1968, p. 546) and modified by Pinder (1970, p. 12). These parameters are used cyclically in the solution process to speed convergence.

Next, the iterative alternating-direction implicit (IADI) method is invoked to solve simultaneously for head changes in the two aquifers. Head changes for the aquifer being considered are assumed to be unknown and head changes for the other aquifer are taken as the most recently computed values during the solution process. Programing of the IADI procedure for solving equations similar to (3) and (4) is described in the "sandstone aquifer" report (McLeod, 1975).

This procedure is continued, alternating solutions between the two aquifers, until the largest discrepancy between row and column computations in each aquifer is less than a prescribed maximum value.

The method then steps to the next time increment and the process is repeated. A flow chart for the program is shown in figure 4.

The solution process is stepped through time in a geometric progression using a multiplying factor of 1.5. This scheme is efficient because small time increments are used during early periods of pumping when head changes are rapid and larger time increments are used during later periods when head changes occur more slowly.

The program permits modeling (1) constant-head or barrier-boundary conditions in both the upper and sandstone aquifers, (2) areal variations in transmissivity and storage coefficients in the sandstone aquifer, (3) areal variations in hydraulic conductivity, saturated thickness, and specific yield in the upper aquifer, and (4) areal variations in leakage coefficient values.

Truncation errors are inherent in the finite-difference equations and are greatest when either nonhomogenous conditions are modeled or the grid spacing in the finite-difference grid is not uniform. These errors result from neglecting

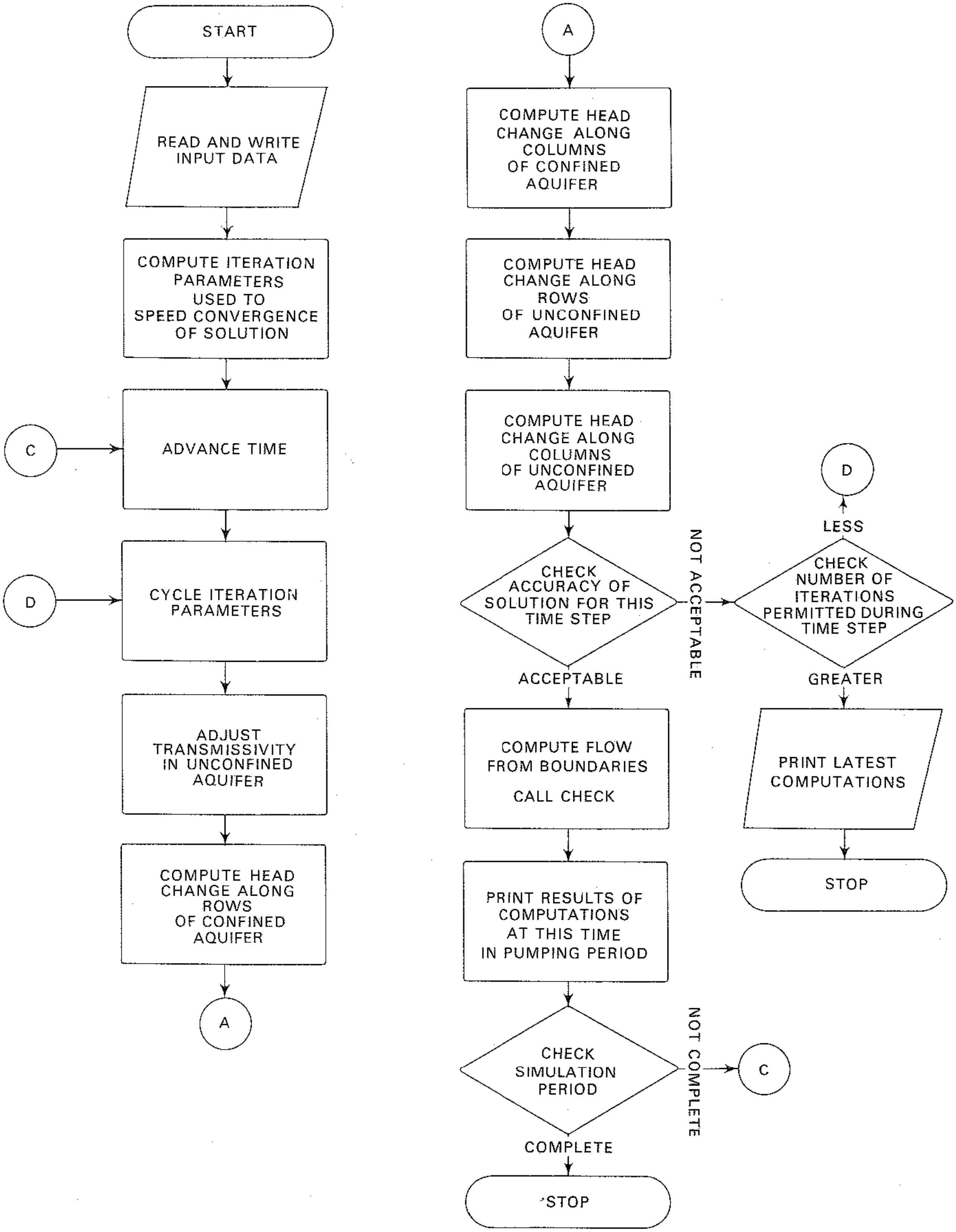


Figure 4. Flow chart for computer program.

(truncating) higher order terms of the finite-difference approximations to the continuous variables in the differential equations. Truncation errors are controlled, in part, by selecting a grid such that the increase in grid size between adjacent elements is no greater than a factor of two, and by avoiding large increases in transmissivity values between adjacent elements.

APPLICATION OF THE PROGRAM TO THE AQUIFER SYSTEM IN DANE COUNTY

Hydrogeology of Dane County

Dane County is underlain by rocks of Precambrian, Cambrian, Ordovician, and Quaternary ages. A stratigraphic column representing the sequence of rocks in the county is given in table 1.

Dense crystalline rock of Precambrian age forms the basement upon which younger geologic units were deposited. The depth below land surface to the Precambrian rocks ranges from less than 600 ft (183 m) to more than 1,300 ft (396 m). Crystalline rock thickness is unknown but is very great.

Cambrian formations overlie the Precambrian bedrock and include, in ascending order, Mount Simon, Eau Claire, Galesville, and Franconia Sandstones, and the Trempealeau Formation. These rocks are chiefly sandstone intermixed with layers of shale, siltstone, and dolomite. The combined average thickness of the Cambrian units within the county, as determined from structure-contour maps, is about 800 ft (244 m); the greatest thickness, about 1,100 ft (335 m), occurs in the southwest.

Ordovician rocks overlie the Cambrian sandstone and include, in ascending order, the Prairie du Chien Group, St. Peter Sandstone, Platteville and Decorah Formations, and Galena Dolomite. The Prairie du Chien Group and the Galena-Platteville unit (Galena Dolomite, and Decorah and Platteville Formations, undifferentiated) generally are massive dolomite. The St. Peter Sandstone consists mostly of sandstone. In many parts of the county the Ordovician units have been removed by erosion. The thickest deposits within the county, more than 500 ft (152 m), occur in the southwest.

Unconsolidated deposits of Quaternary age overlie bedrock of Cambrian and Ordovician age. These include morainal deposits, outwash, and glacial-lake deposits ranging in thickness from zero to about 370 ft (113 m). These deposits are covered in places by thin loess, alluvium, or marsh deposits.

A more comprehensive discussion of the geology of Dane County is available in Cline's report (1965).

The aquifer system underlying Dane County is composed of the entire thickness of Cambrian, Ordovician, and Quaternary deposits. The Precambrian basement rocks are relatively impermeable and are assumed to form the base of the system.

The aquifer system is subdivided into the sandstone aquifer and the upper aquifer (table 1). This subdivision is based on well-construction practices in the county which, in turn, reflect hydrologic conditions.

Table 1.--Generalized stratigraphy and aquifer system in Dane County, Wisconsin.

Era or System	Geologic unit		Dominant lithology	Hydrologic unit	Saturated thickness (ft)
Quaternary	Holocene and Pleistocene deposits		Clay, silt, sand	Upper aquifer	50-450
Ordovician	Galena Dolomite		Dolomite		
	Decorah Formation		Dolomite		
	Platteville Formation		Dolomite		
	St. Peter Sandstone		Sandstone		
	Prairie du Chien Group		Dolomite		
Cambrian	Trempealeau Formation		Sandstone and dolomite		
	Franconia Sandstone	Reno Member*	Sandstone		
		Mazomanie Sandstone Member			
		Ironton Sandstone Member		Sandstone aquifer	450-900
	Galesville Sandstone		Sandstone		
	Eau Claire Sandstone		Sandstone and shale		
	Mount Simon Sandstone		Sandstone		
Precambrian	Precambrian rocks undifferentiated		Crystalline rocks	Not an aquifer	

*The Reno Member of the Franconia Formation is herein adopted by the U.S. Geological Survey according to the usage of the University of Wisconsin Geological and Natural History Survey.

The Ironton Sandstone Member of the Franconia Sandstone, plus the Galesville, Eau Claire, and Mount Simon Sandstones, collectively form the sandstone aquifer. The sandstone aquifer generally is composed of coarse- to fine-grained sandstone. Ground-water movement is primarily through the intergranular pore spaces of the sandstone, with secondary movement through fractures and joints. The areal variation in saturated thickness of this aquifer, as determined from structure-contour maps, is shown by figure 5.

The Mazomanie Sandstone Member and Reno Member of the Franconia Sandstone, the Trempealeau Formation, plus all Ordovician and Quaternary deposits, collectively form the upper aquifer (table 1). Ground-water movement in the upper aquifer is probably through fractures, joints, and solution channels in dolomitic rocks; through intergranular pore spaces, fractures, and joints in sandstone; and through intergranular pore spaces in unconsolidated deposits. The areal variation in saturated thickness of this aquifer, as determined from a structure contour map of the base of the aquifer and a water-table map, is shown by figure 6.

The Mazomanie Sandstone and Reno Members of the Franconia Sandstone, the Trempealeau Formation, and Quaternary deposits form the major part of the saturated thickness of the upper aquifer. These deposits are saturated in part throughout most of the county.

The Prairie du Chien Group and St. Peter Sandstone generally form a small part of the saturated thickness of the upper aquifer. These deposits are generally saturated in part where they are present, but they have been eroded away in many areas. Their saturated thickness is greatest in the southwest part of the county.

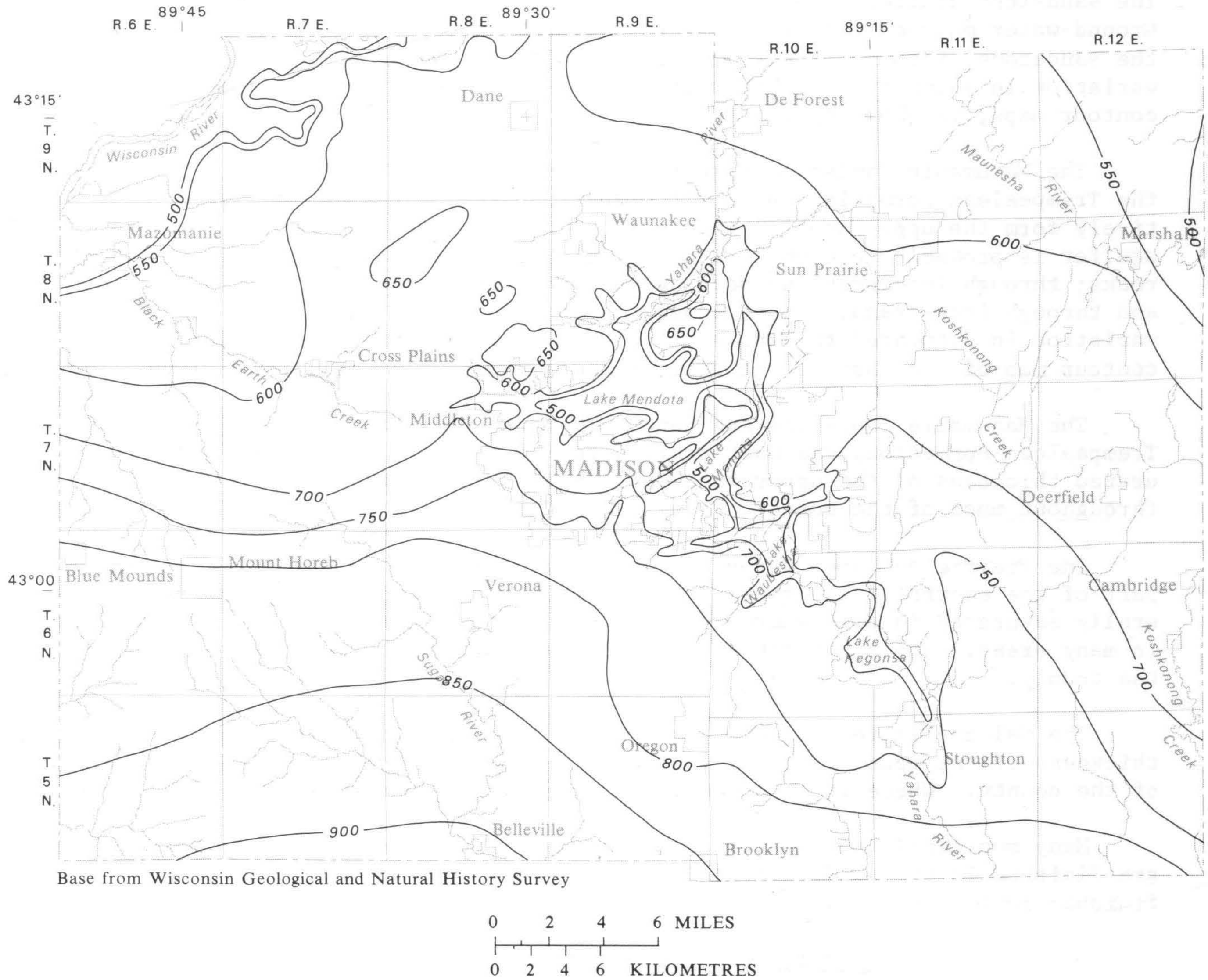
The Galena-Platteville unit does not form a significant part of the saturated thickness of the upper aquifer. The unit has been eroded away throughout much of the county. Where it is present, it is generally unsaturated.

Many municipal and industrial wells penetrate to Precambrian bedrock and are finished in the sandstone aquifer. Most domestic wells in the county are finished in the upper aquifer.

Hydrologic Changes Caused by Pumping

Hydrologic changes that have occurred because of pumping from the sandstone aquifer include declines in ground-water levels in both the sandstone and upper aquifers and reductions in base flow to streams. Declines in water levels in the sandstone aquifer reflect flow changes that occurred there to adjust to pumping. Declines in water levels in the upper aquifer reflect flow changes that occurred there to adjust to leakage changes between the sandstone and upper aquifers caused by pumping. Reductions in streamflow reflect flow changes that occurred between surface-water bodies and the aquifer system that are attributed to discharge from the system caused by pumping.

Declines in ground-water levels in the sandstone aquifer since pumping began ranged from less than 10 ft (3 m) to more than 70 ft (21 m) (fig. 7) by 1970. These head declines were determined from water-level data collected for municipal and industrial wells. Largest head declines, more than 70 ft in places (21 m), occurred along a northeast-southwest line through the Madison area. This trend was due partly to pumpage distribution and partly to the hydrogeology of the aquifer system.

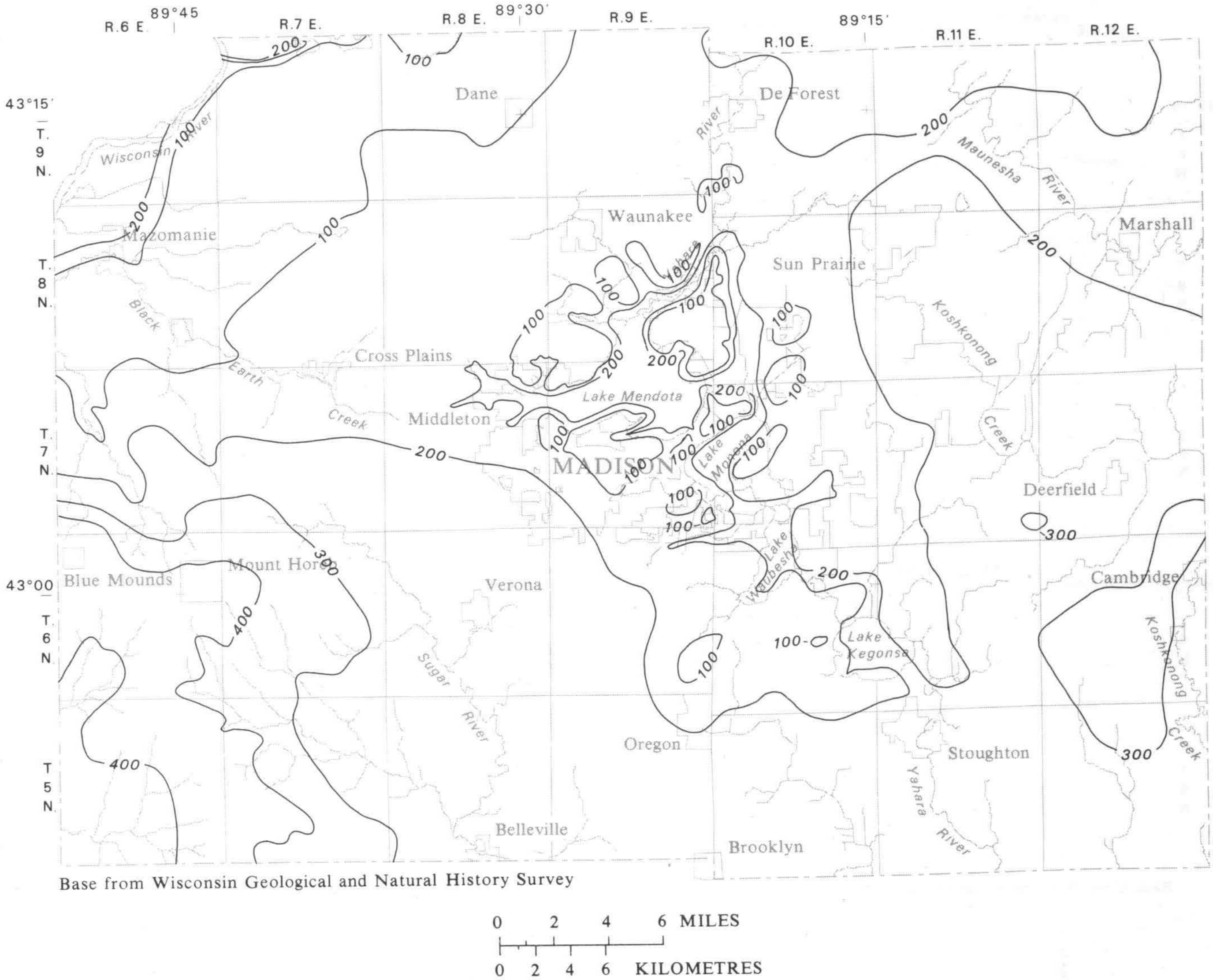


EXPLANATION

——700—— Line of approximate equal saturated thickness of the sandstone aquifer

Interval 50 and 100 feet (15 and 30 metres)

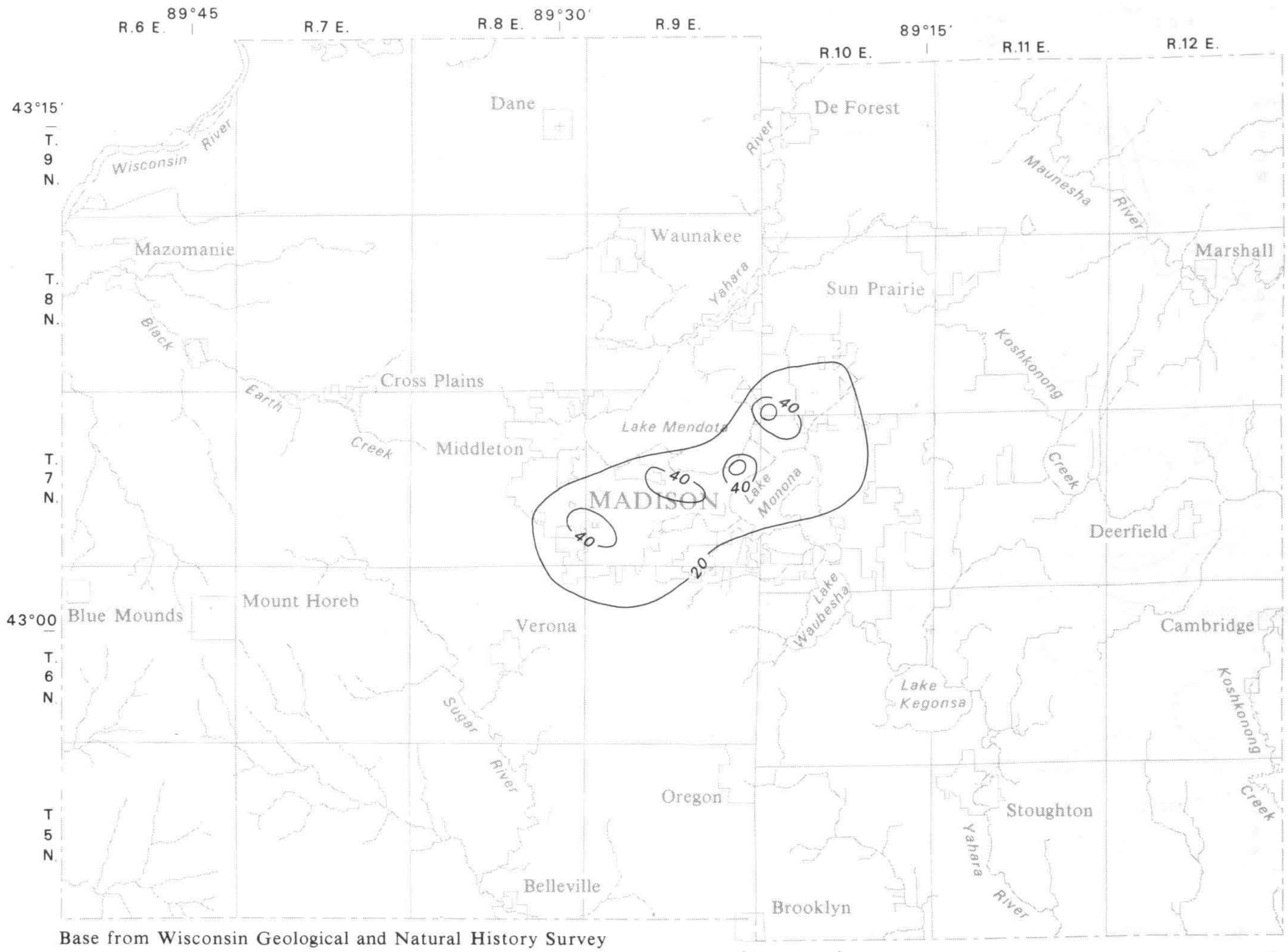
Figure 5. Saturated thickness of the sandstone aquifer.



EXPLANATION

——— 100 ——— Line of approximate equal saturated thickness before development of sandstone aquifer
Interval 100 feet (30 metres)

Figure 6. Saturated thickness of the upper aquifer.



Base from Wisconsin Geological and Natural History Survey

0 2 4 6 MILES

0 2 4 6 KILOMETRES

EXPLANATION

——— 20 ——— Line of approximate equal head decline

Interval 20 feet (6 metres)

Figure 7. Declines in water levels in the sandstone aquifer by 1970.

Declines in water levels in the upper aquifer by 1970 ranged from 0 to about 20 ft (6 m) (fig. 8). These head declines were estimated from water-level measurements in shallow wells. Largest head declines, about 20 ft (approximately 6 m), occurred in central Madison. Head declines in the upper aquifer generally were much smaller than head declines in the sandstone aquifer at corresponding locations.

Reductions in streamflow due to pumping are not readily apparent on streamflow hydrographs. However, it has been demonstrated using double-mass curve techniques that yearly reductions in streamflow of the Yahara River past the stream-gaging station (fig. 9) could be approximated by the yearly amounts of water exported from the basin (K. B. Young, written commun., 1968).

The reduction in streamflow of the Yahara River by 1970 at the stream-gaging station (fig. 9) was estimated for purposes of this report to be between 46 and 57 ft^3/s (1.3 and 1.6 m^3/s). The 46 ft^3/s (1.3 m^3/s) is the 1970 average amount of water exported from the upper Yahara River basin by the Madison Metropolitan Sewerage District (fig. 9). This water was pumped from municipal and industrial wells in the basin. The 57 ft^3/s (1.6 m^3/s) is the 1970 average amount of ground water pumped from municipal and industrial wells in the basin. This range of values allows for an accounting of pumpage lost to evapotranspiration or other consumptive uses not detected by the double-mass curve analysis.

Modeling the Aquifer System

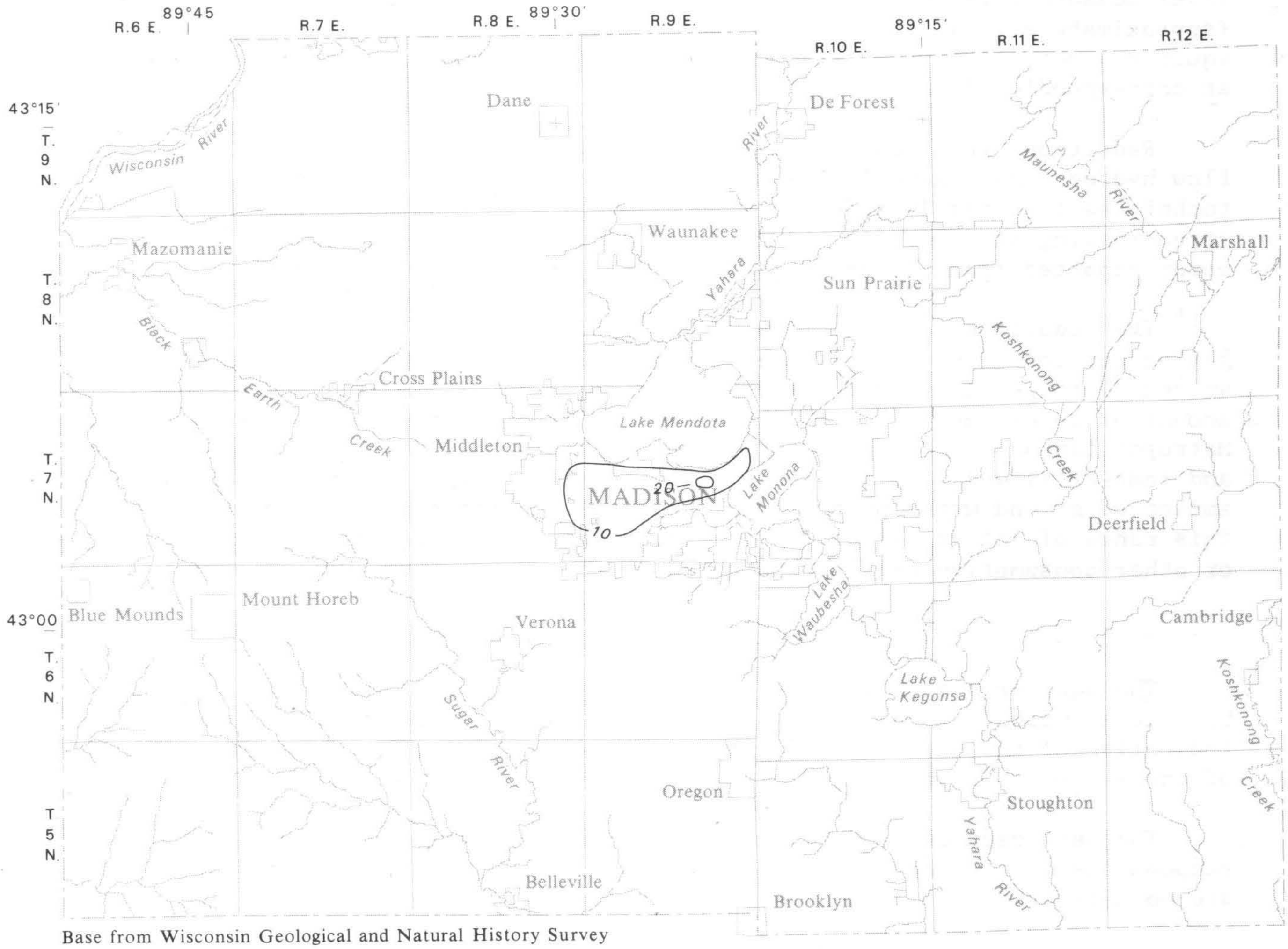
The aquifer system was modeled as a confined sandstone aquifer overlain by a leaky upper aquifer and underlain by impermeable bedrock. This model is indicative of the overall characteristics of the system based on the response of the system to pumping from the sandstone aquifer.

The heterogeneous and ansiotropic properties of the upper aquifer are responsible for the confinement of the underlying sandstone aquifer. There are no definitive confining beds directly overlying the sandstone.

The confining bed characteristics of the upper aquifer were approximated by a leakage coefficient (p. 6). Areal variations in leakage coefficient were computed by the program using the ratio of apparent vertical hydraulic conductivity to initial saturated thickness assigned to each upper aquifer element.

Two types of boundaries were approximated for the model: constant-head boundaries and boundaries across which no flow could occur. Perennial wetlands, streams, and lakes were assumed to be constant-head boundaries for the upper aquifer. The Wisconsin River to the northwest was modeled as a constant-head boundary also in the sandstone aquifer. An outcrop of Precambrian crystalline rock just east of Dane County (fig. 10) was assumed to retard ground-water movement in that area and was modeled as a no-flow boundary in both the upper and sandstone aquifers. Both aquifers were modeled as being infinite in areal extent in remaining areas by extending the boundary edges of the finite-difference grid to such large distances that the area of interest would not be affected by the boundary edges during the period of analysis.

The finite-difference grid used for this model is identical to the grid used in the sandstone aquifer model (McLeod, 1975). Grid spacing is small



Base from Wisconsin Geological and Natural History Survey

0 2 4 6 MILES

0 2 4 6 KILOMETRES

EXPLANATION

——— 10 ——— Line of approximate head decline.

Interval 10 feet (3 metres)

Figure 8. Declines in water levels in the upper aquifer by 1970.

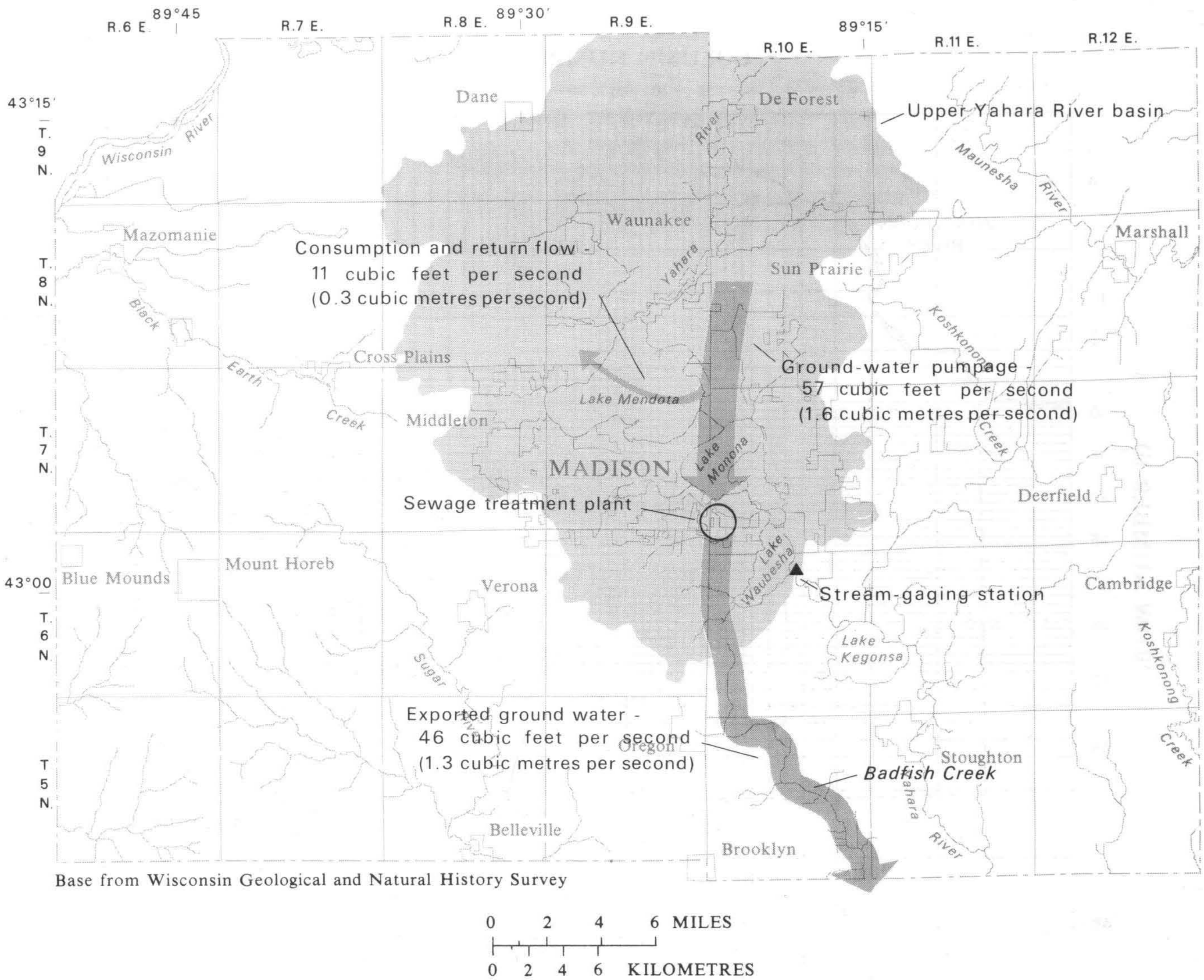


Figure 9. Relation between ground-water pumpage and exported water for upper Yahara River, 1970.

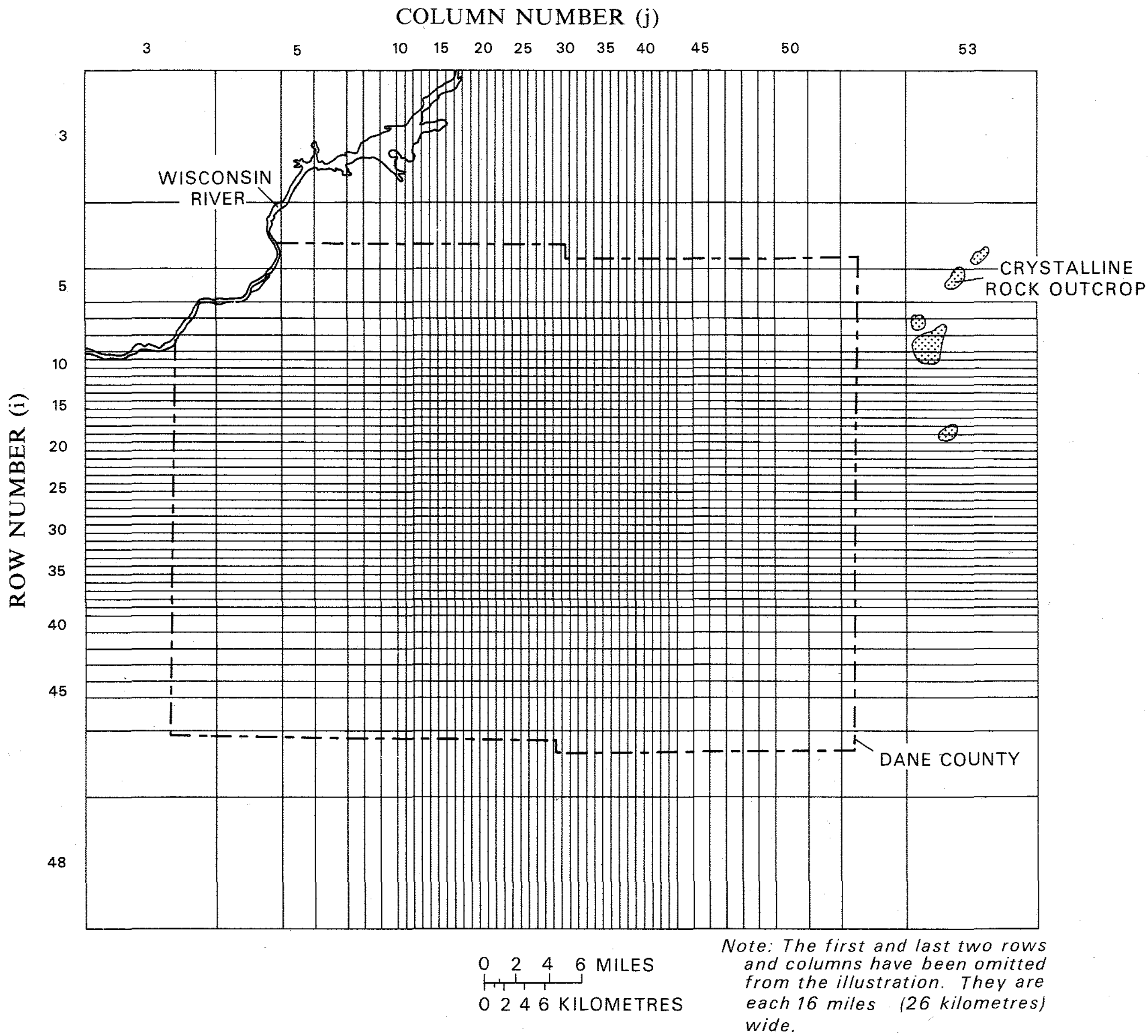


Figure 10. Finite-difference grid used for modeling the aquifer system.

enough to reasonably represent the aquifer system in the Madison area by averaging physical properties within grid elements. The finite-difference grid configuration is shown by figure 10.

Steady-state analyses of flow changes were used for modeling the aquifer system. This type of analysis assumes that water levels stabilize immediately in response to pumping changes and that effects of past pumping on current water levels could be neglected.

Several observations suggest that the errors associated with using steady-state analyses could be neglected. First, water levels in wells stabilize after relatively short periods of pumping. Second, estimated reductions in streamflow past the stream-gaging station (fig. 9) have kept pace with ground-water pumpage being exported from the basin. And third, aquifer-test data suggest that the time required for pumping wells to reach steady state at test locations would be approximately 1 year.

The proximity of pumping wells to streams, lakes, and perennial wetlands and the distribution of these surface features largely accounts for the ability of water levels to stabilize rapidly when responding to pumping changes. When a new well is put into production, or the pumping schedule of an existing well is changed, the hydrologic equilibrium of the aquifer system is influenced by the new discharge from the system, water levels in the aquifer fall, and water is withdrawn from storage in the system. A new equilibrium cannot be reached until water levels stabilize at a lower level and there is no further loss from storage. A new equilibrium occurs after sufficient time has elapsed for the system to reduce natural discharge to nearby surface waters by an amount equal to pumpage changes.

Approximating Areal Variations in Physical Properties of the Aquifer System

Physical properties needed for a steady-state model of the aquifer system are horizontal hydraulic conductivity, initial saturated thickness, and apparent vertical hydraulic conductivity values for the upper aquifer, and transmissivity for the sandstone aquifer. These properties are not areally uniform.

Areal differences in horizontal hydraulic conductivity of the upper aquifer (fig. 11) are based on areal differences in geology. The upper aquifer is composed of thick unconsolidated deposits in the deeply buried preglacial Yahara and Wisconsin River valleys. A hydraulic conductivity value of 260 $(gal/d)/ft^2$ (10.6 m/d) was used for these areas. This value was estimated using well-log data. Remaining areas of the upper aquifer are composed mostly of bedrock. A hydraulic conductivity value of 130 $(gal/d)/ft^2$ (5.5 m/d) was used for these areas. This value was determined using a method outlined by Jenkins (1963). The method involved a graphical multiple-regression analysis that estimated average hydraulic conductivities for the upper aquifer and the sandstone aquifer from well-log and aquifer-test data.

Areal variations in saturated thickness of the upper aquifer (fig. 6) were determined as outlined in the section on Hydrogeology of Dane County.

Base from Wisconsin Geological and Natural History Survey

0 2 4 6 MILES

0 2 4 6 KILOMETRES

EXPLANATION

Areas where the upper aquifer is composed entirely of unconsolidated deposits

Hydraulic conductivity = 260 gpd/ft^2 (10.6 m/d)

Areas where the upper aquifer is not composed entirely of unconsolidated deposits

Hydraulic conductivity = 130 gpd/ft^2 (5.3 m/d)

Figure 11. Horizontal hydraulic conductivity in the upper aquifer.

Areal differences in apparent vertical hydraulic conductivity (fig. 12) for the upper aquifer were available from the sandstone aquifer report. These values were based on areal differences in geology. One set of average values was estimated for areas where the upper aquifer was composed of thick unconsolidated deposits. Another set of average values was estimated for remaining areas of the aquifer system.

Areal differences in transmissivity for the sandstone aquifer (fig. 13) also were available from the sandstone aquifer report. These values were estimated using the average hydraulic conductivity value for the aquifer [55 (gal/d)/ft^2 or 2.2 m/d] determined by the Jenkins method and the thickness map (fig. 5) for the aquifer.

Calibrating the Model

A steady-state analysis using 1970 average daily pumpage data (fig. 14) was used to calibrate the model.

Hydrologic changes computed by the program in the first calibration run and observed changes in the aquifer system were in general agreement. The overall pattern of head declines computed by the model (figs. 15 and 16) reproduced the observed pattern fairly well (figs. 7 and 8). There were, however, some differences between observed and computed head declines in local areas. These differences could reflect inadequate information to define in detail head declines that have occurred. The computed reduction in average annual flow of the Yahara River past the stream-gaging station (fig. 9) was 51.9 ft^3/s (1.47 m^3/s). This value fell well within the estimated limits of 46 to 57 ft^3/s (1.3 to 1.6 m^3/s). These results indicated that no adjustments to the model were required.

The model computed a net diversion of approximately 5 ft^3/s (about 0.1 m^3/s) of water from adjacent basins to the upper Yahara River basin during 1970. This quantity is the difference between pumping in the basin and the computed reduction in streamflow at the gaging station.

This value appears to be reasonable. Head declines in the sandstone and upper aquifers have been large enough to influence water movement near the upper Yahara River basin divide in areas of the basin.

Computing Future Hydrologic Declines

Rates and locations of future pumping from the sandstone aquifer had to be determined to compute head declines by 1980 and 1990. Pumping rates were determined by the author using an arithmetic projection of the past pumping trend for each user to the year 1990. Each user's long-range ground-water development plan then was used to distribute the projected pumpages. For those users having no long-range plan, projected pumping was distributed among wells in use in 1970. The rates and locations of pumping from the sandstone aquifer by 1980 and 1990, using the above methods, are shown in figures 17 and 18, respectively. A summary of the projected pumping for each user is given in table 2 along with the 1970 values.

Base from Wisconsin Geological and Natural History Survey

0 2 4 6 MILES

0 2 4 6 KILOMETRES

EXPLANATION

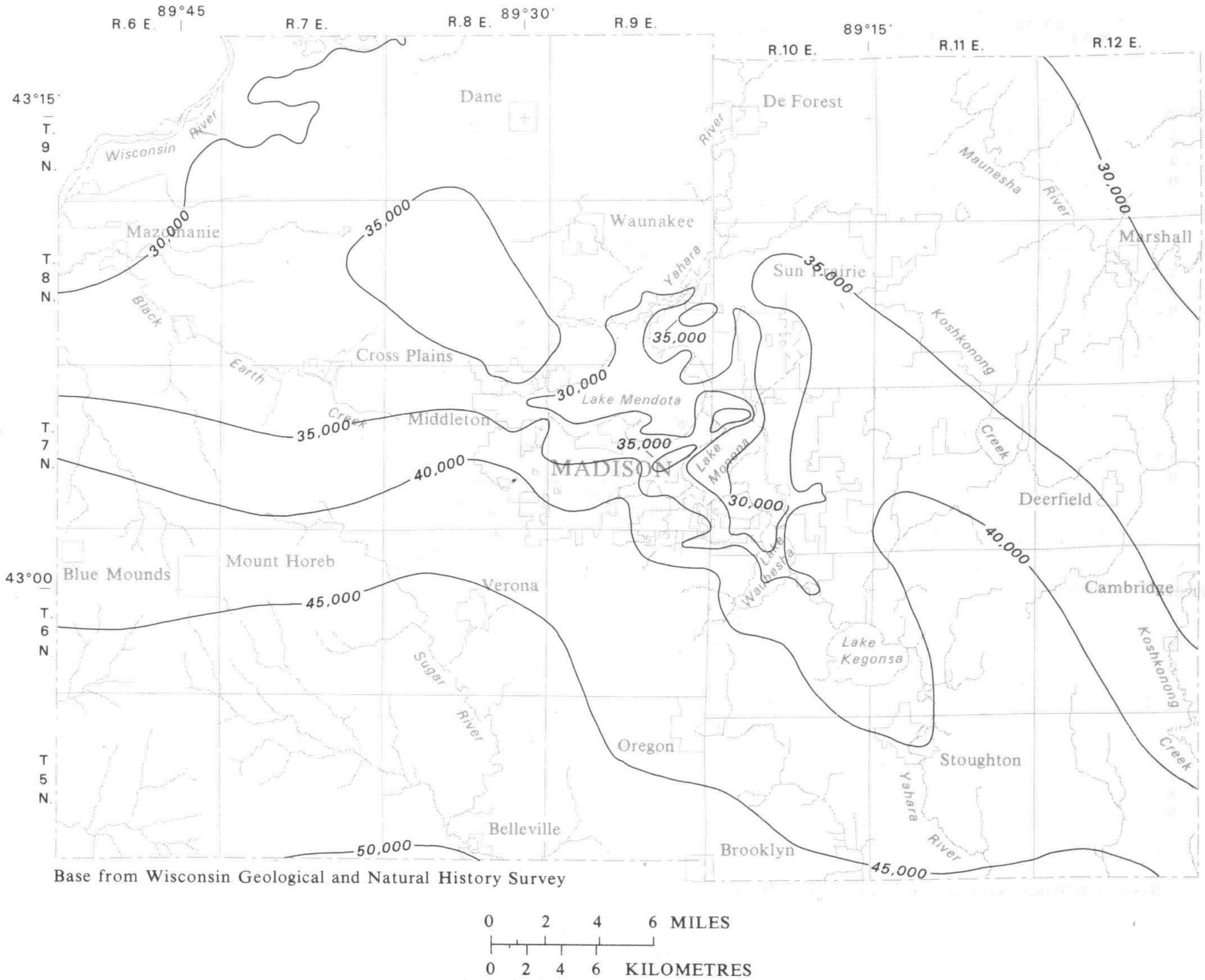
Areas where the upper aquifer is composed entirely of unconsolidated deposits

Vertical hydraulic conductivity = 0.2 gpd/ft (0.002 m^2/d)
Specific storage = 0.0002 ft (0.0007/m)

Areas where the upper aquifer is not composed entirely of unconsolidated deposits

Vertical hydraulic conductivity = 0.01 gpd/ft (0.0001 m^2/d)
Specific storage = 0.000004 /ft (0.00001 /m)

Figure 12. Apparent vertical hydraulic conductivity in the upper aquifer.

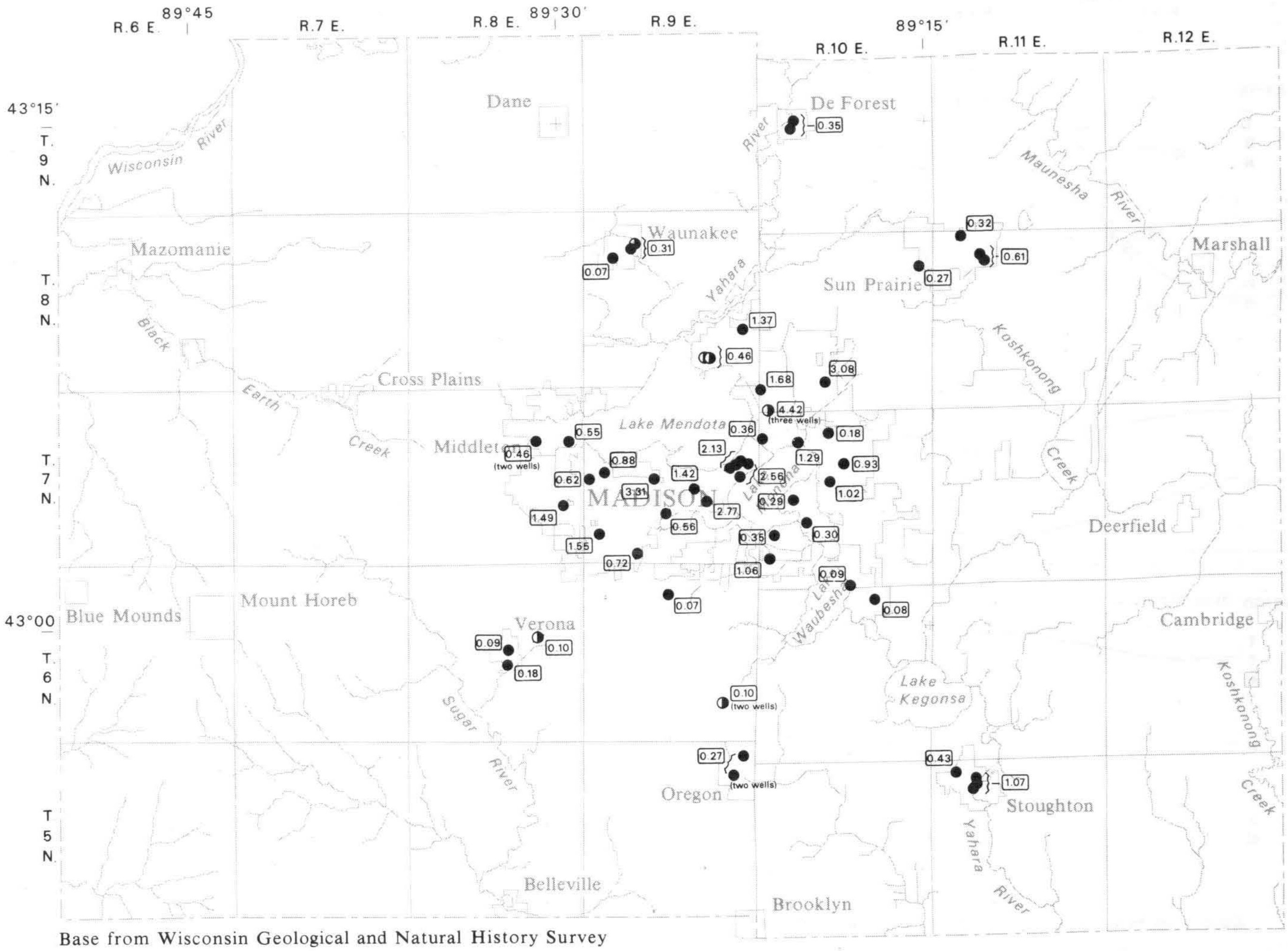


EXPLANATION

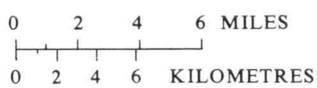
——— *40,000* ——— Line of approximate equal transmissivity

Interval 5,000 gpd/ft (62 m^2/d)

Figure 13. Transmissivity in the sandstone aquifer.



Base from Wisconsin Geological and Natural History Survey



EXPLANATION

1.37 Average daily pumpage, in million gallons per day

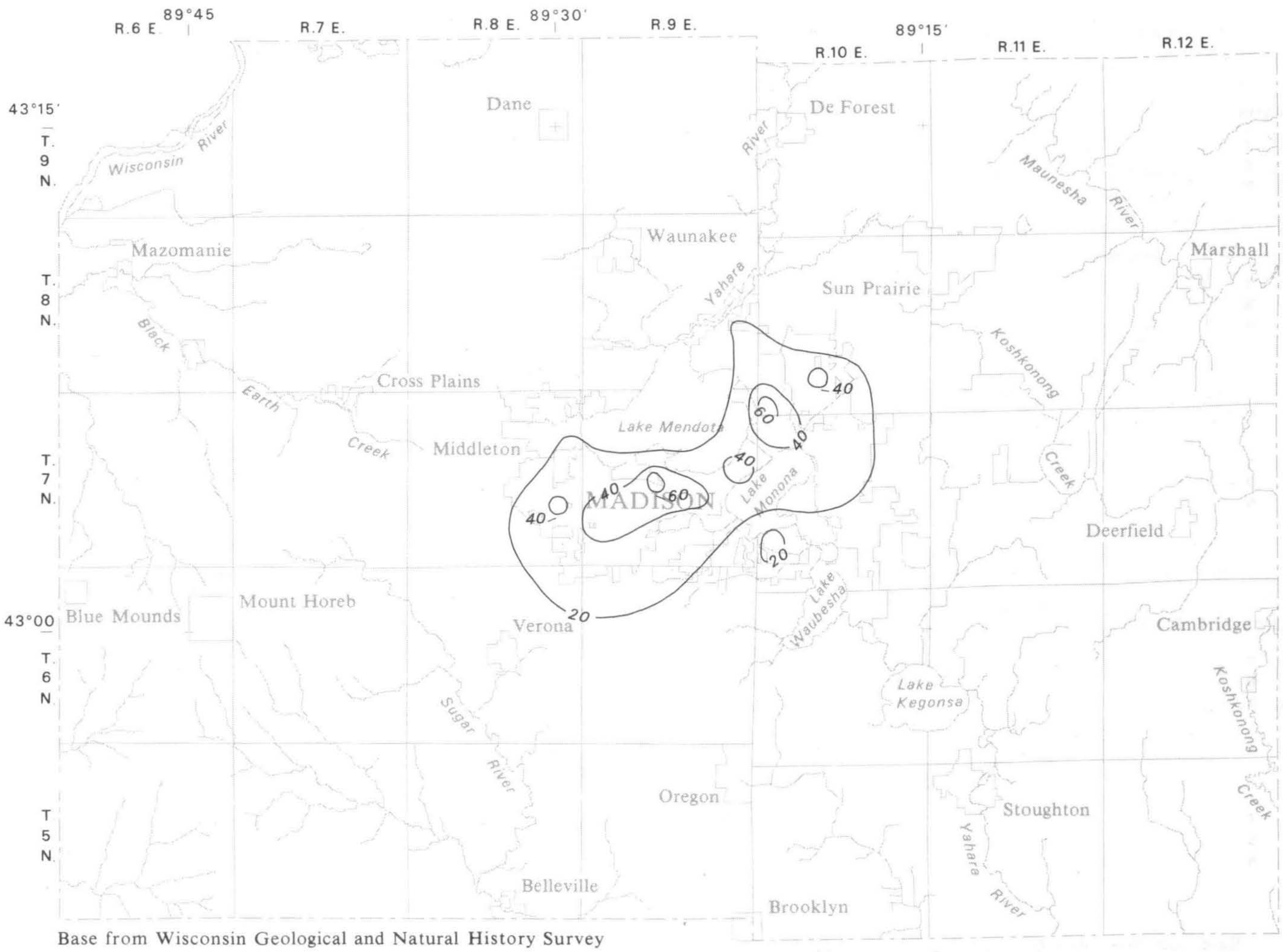
}-0.35 Combined pumpage of wells
Wherever two or more wells are assigned to pump from the same finite-difference grid element

● Water utility well

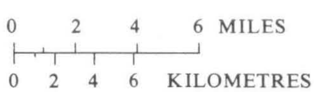
◑ Industrial or public institution well

(two wells) Number of wells represented by well symbol, if more than one well

Figure 14. Pumpage from the sandstone aquifer during 1970.



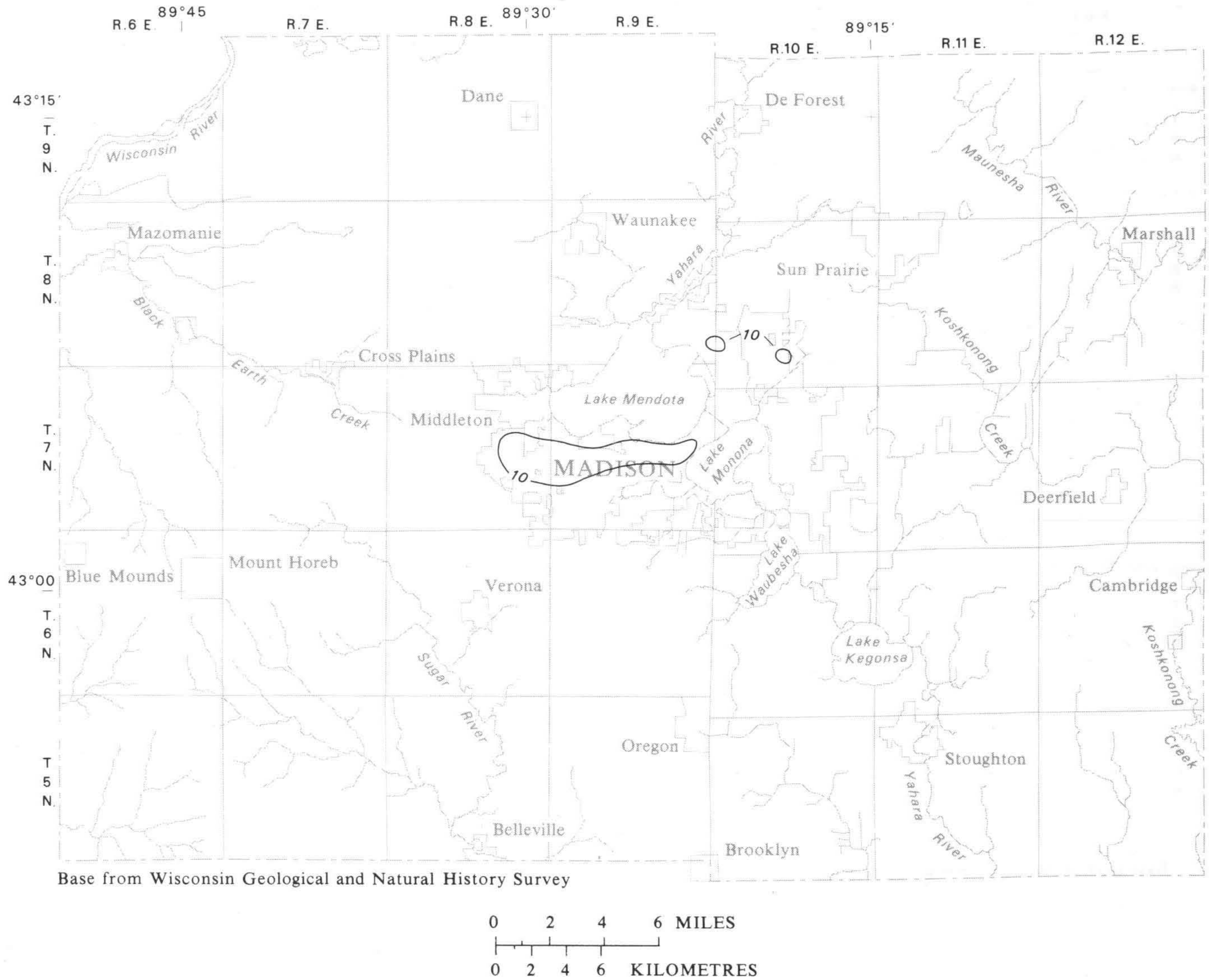
Base from Wisconsin Geological and Natural History Survey



EXPLANATION

———20——— Line of approximate equal head decline
Interval 20 feet (6 metres)

Figure 15. Computed head declines in the sandstone aquifer by 1970.

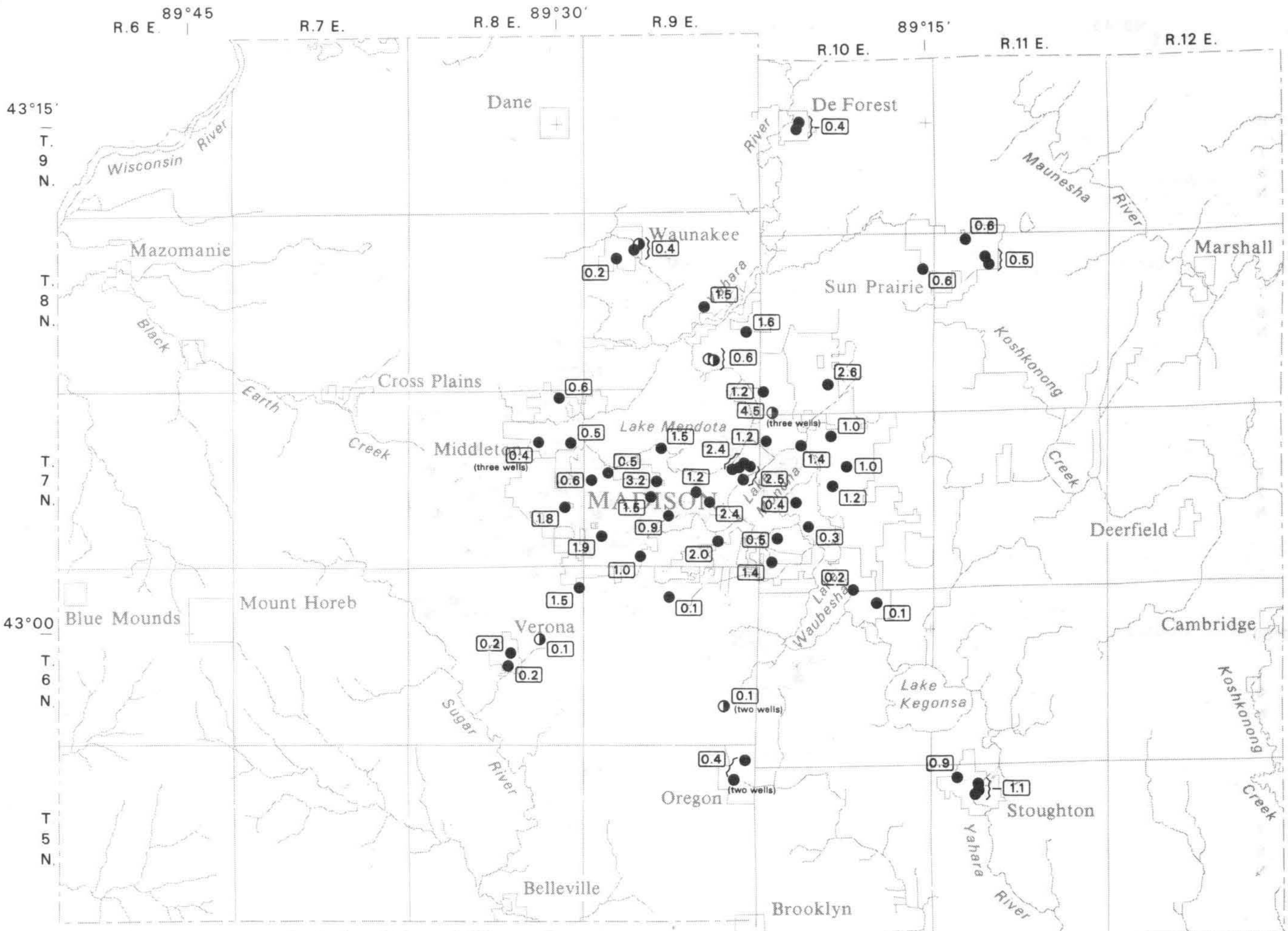


EXPLANATION

——— 10 ——— Line of approximate equal head decline

Interval 10 feet (3 metres)

Figure 16. Computed head declines in the upper aquifer by 1970.



Base from Wisconsin Geological and Natural History Survey

EXPLANATION

0.4 Average daily pumpage, in million gallons per day

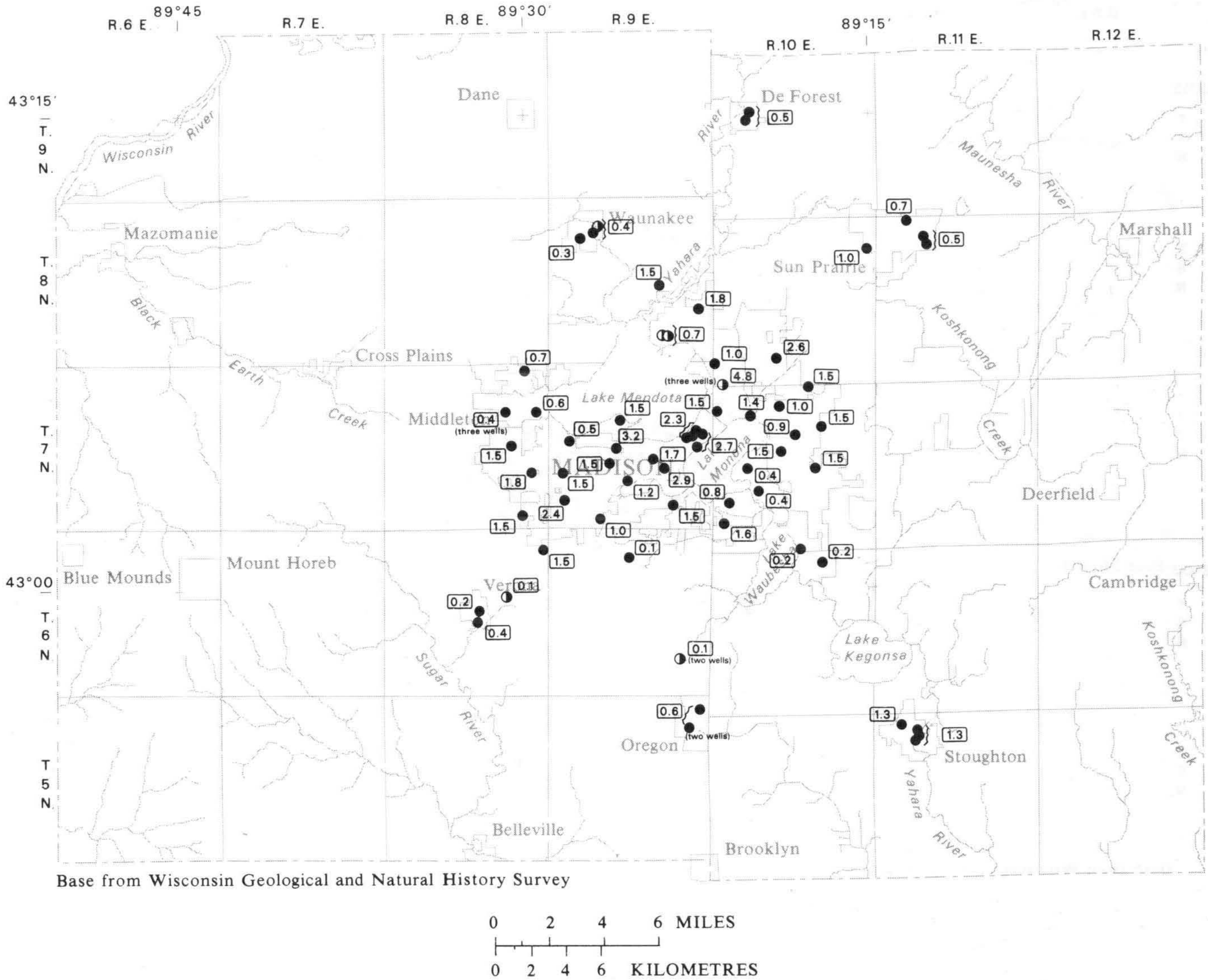
2.5 Combined pumpage of wells
Wherever two or more wells are assigned to pump from the same finite-difference grid element

● Water utility well
Existing or proposed

◑ Industrial or public institution well

(three wells) Number of wells represented by well symbol, if more than one well

Figure 17. Estimated pumpage from the sandstone aquifer by 1980.



EXPLANATION

0.4 Average daily pumpage, in million gallons per day

0.5 Combined pumpage of wells
Wherever two or more wells are assigned to pump from the same finite-difference grid element

● Water utility well
Existing or proposed

◑ Industrial or public institution well

(three wells) Number of wells represented by well symbol, if more than one well

Figure 18. Estimated pumpage from the sandstone aquifer by 1990.

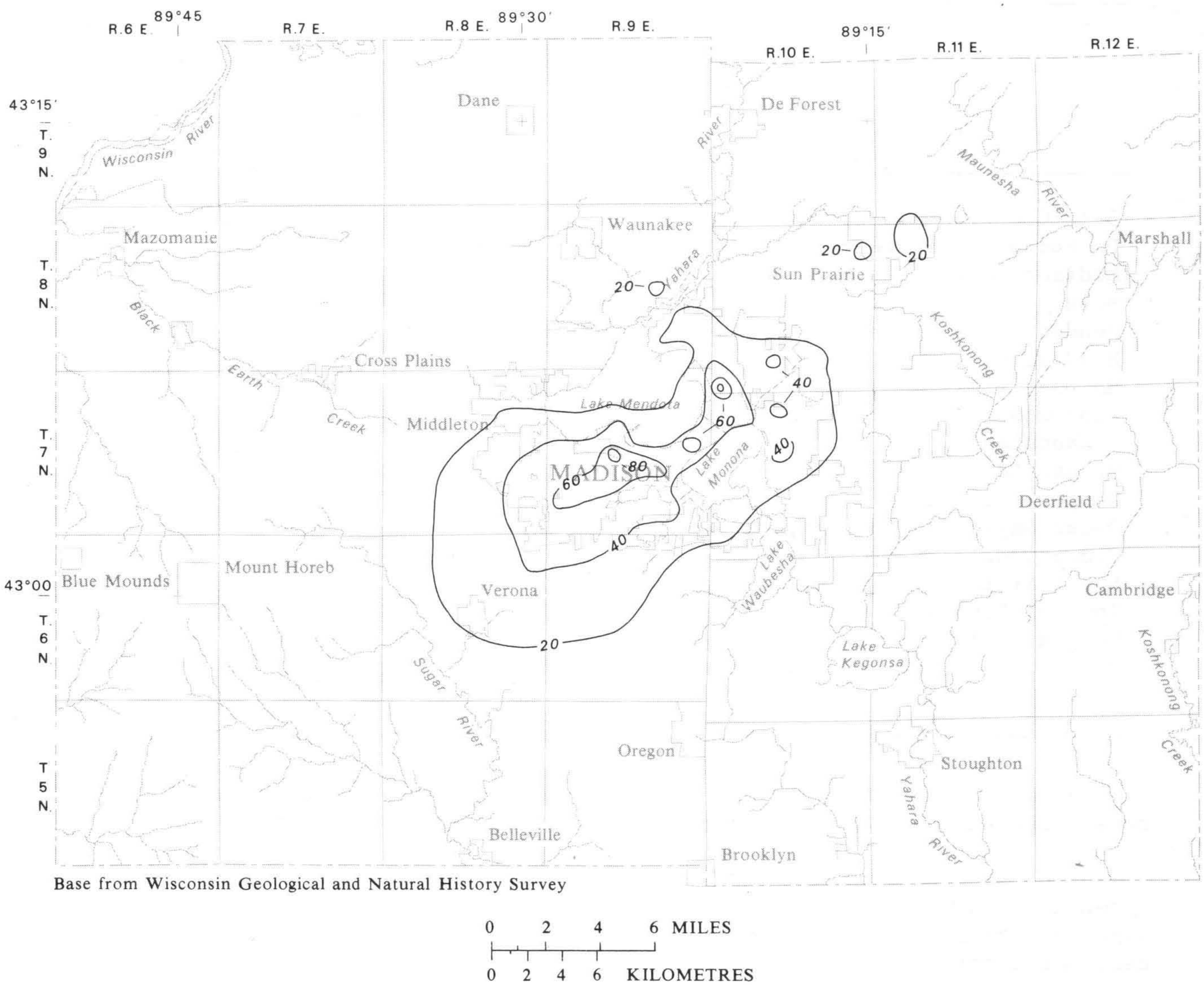
Table 2.--Reported and projected pumpages from the sandstone aquifer.

User	Reported pumpage (Mgal/d)	Projected pumpage (Mgal/d)	
	1970	1980	1990
Dane County Home, Verona	0.10	0.1	0.1
De Forest, village of	.35	.4	.5
Madison Water Utility	28.98	39.0	49.5
McFarland, village of	.17	.3	.4
Mendota State Hospital, Madison	.46	.6	.7
Middleton, city of	1.01	1.5	1.7
Monona, village of	.94	1.2	1.6
Oconomowoc Canning Company, Waunakee	.13	.2	.2
Oregon School for Girls	.10	.1	.1
Oregon, village of	.27	.4	.6
Oscar Mayer and Company, Madison	4.42	4.5	4.8
Stoughton, city of	1.50	2.0	2.6
Sun Prairie, village of	1.20	1.7	2.2
Town of Fitchburg	.07	.1	.1
Verona, village of	.27	.4	.6
Waunakee, village of	.25	.4	.5
Total	40.22	52.9	66.2

Head declines for 1980 and 1990 were computed by running the model twice, once using pumpages for 1980 and once using pumpages for 1990.

The computed 1980 and 1990 head declines in the sandstone aquifer are shown in figures 19 and 20, respectively. These head declines reflect the same general northeast-southwest trend as 1970 head declines. Maximum head declines continued to occur to the east and south of Lake Mendota and on the southwest side of Madison. Head declines in these areas by 1990 may range from 40 to 80 ft (12.2-24.4 m). This would represent an increase over 1970 head declines of 10 to 20 ft (3.0-6.1 m) to the east and south of Lake Mendota and 20 to 40 ft (6.1-12.2 m) on the southwest side of Madison.

The computed 1980 and 1990 head declines in the upper aquifer are shown in figures 21 and 22, respectively. These head declines also follow the same general trend as the 1970 head declines, with maximum declines occurring on the west side of Madison. Declines in this area may range between approximately 10 to 20 ft (3.0-6.1 m) by 1990. This would represent a general increase over 1970 head declines of approximately 5 to 10 ft (1.5-3.0 m) on the west side. Head declines in other areas of Madison by 1990 may be approximately 5 ft (1.5 m) greater than 1970 head declines.

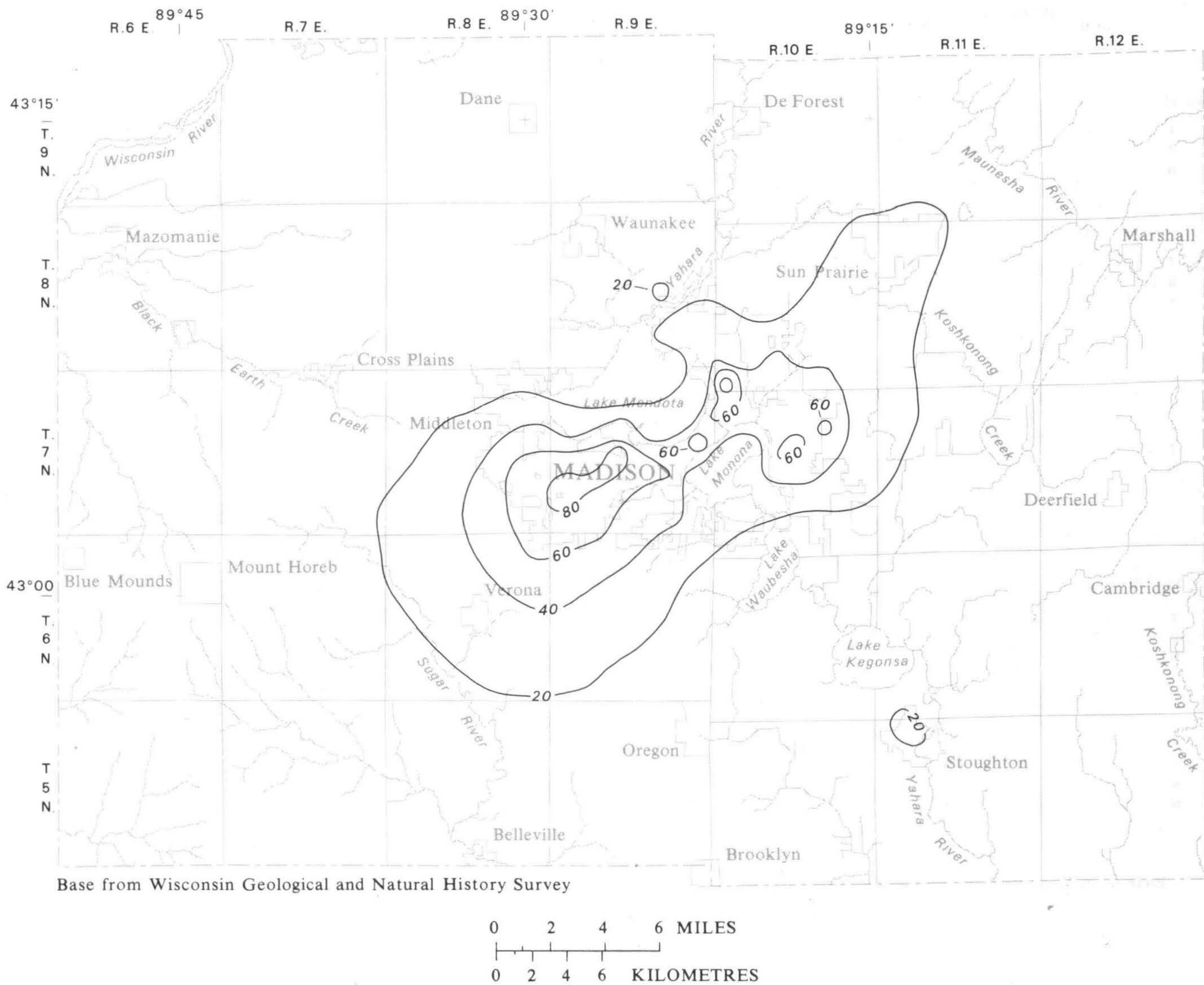


EXPLANATION

———— 20 ———— Line of approximate equal head decline

Interval 20 feet (6 metres)

Figure 19. Computed head declines in the sandstone aquifer by 1980.

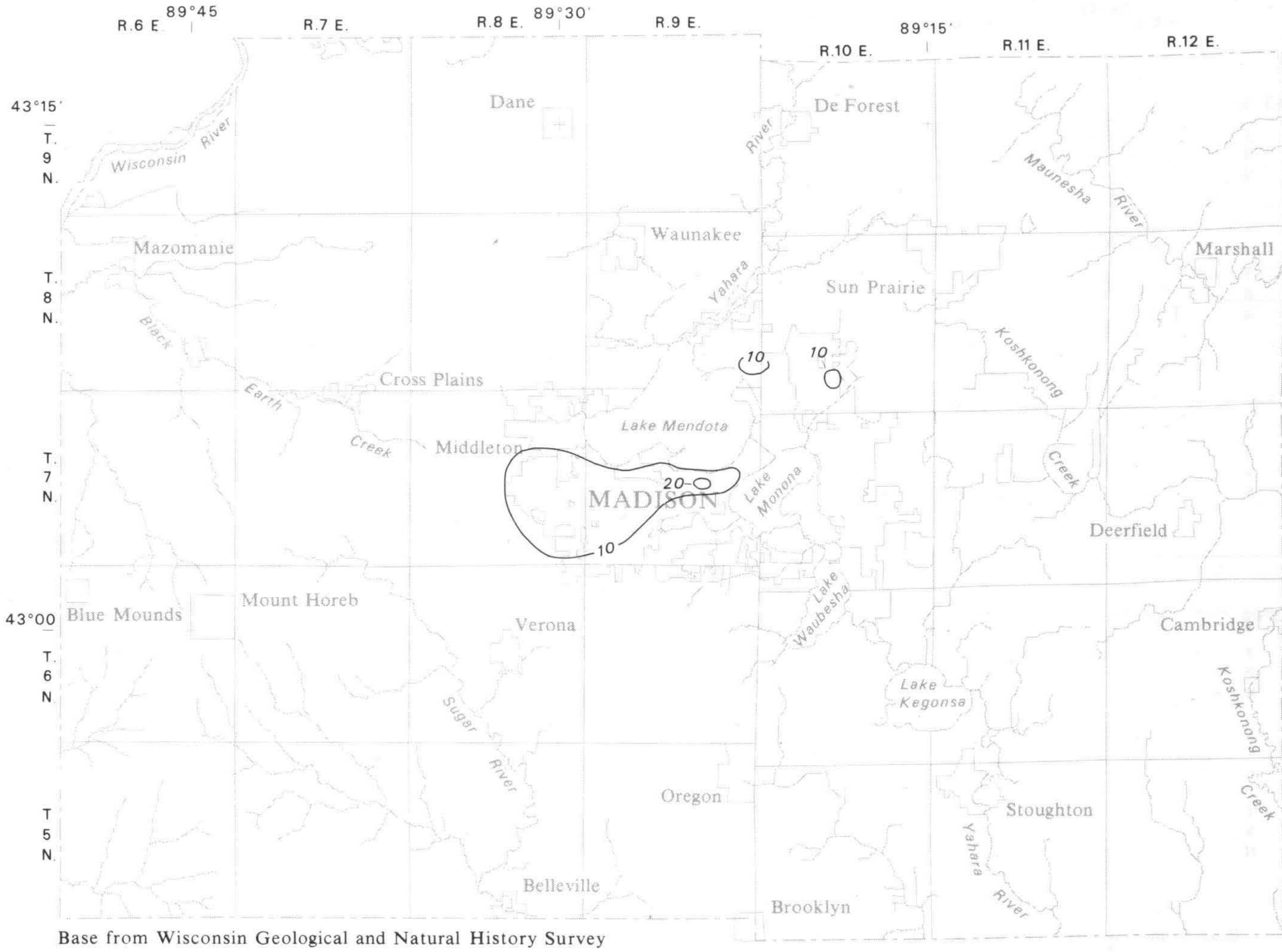


EXPLANATION

———20——— Line of approximate equal head decline

Interval 20 feet (6 metres)

Figure 20. Computed head declines in the sandstone aquifer by 1990.



Base from Wisconsin Geological and Natural History Survey

0 2 4 6 MILES

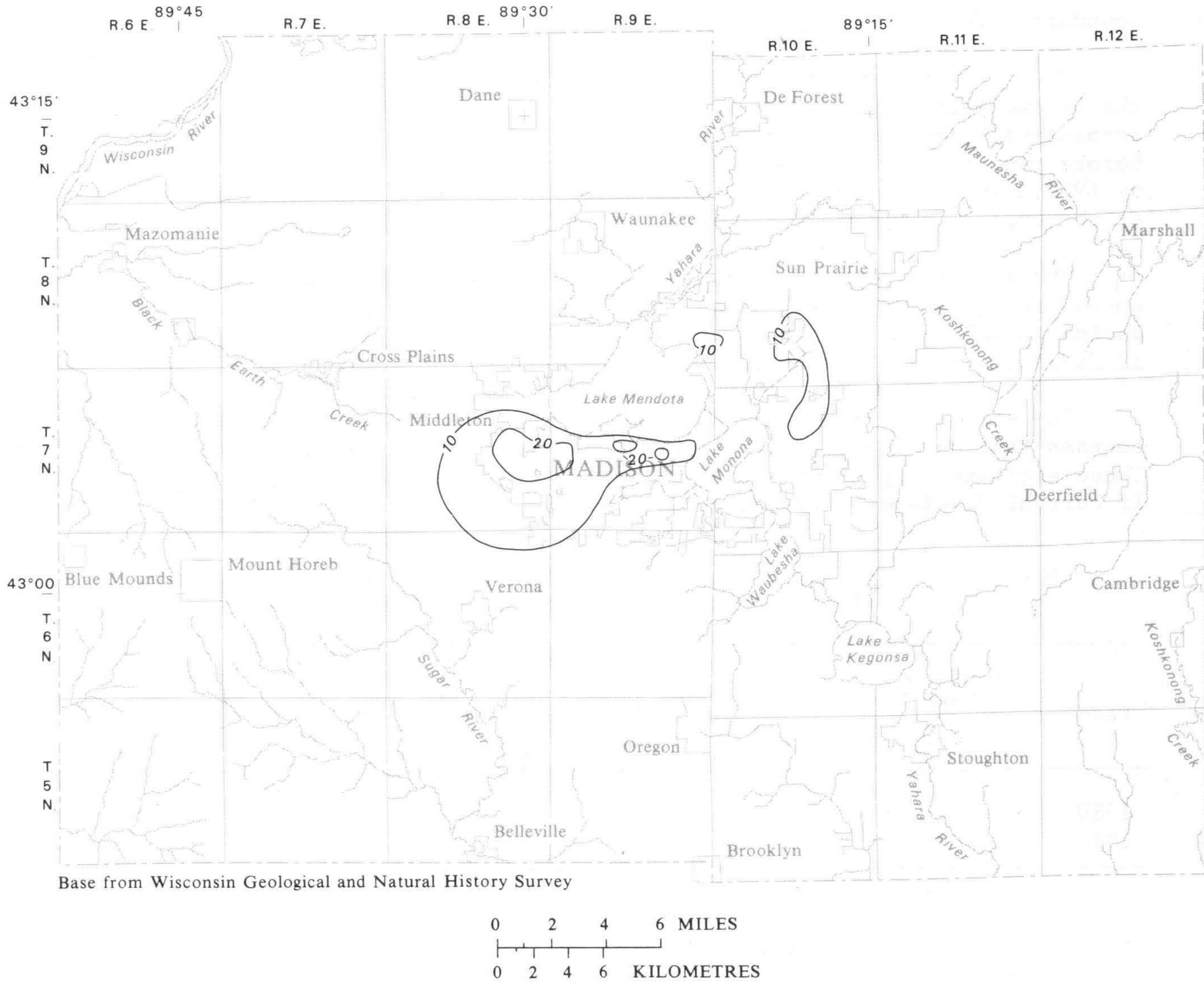
0 2 4 6 KILOMETRES

EXPLANATION

——— 10 ——— Line of approximate equal head decline

Interval 10 feet (3 metres)

Figure 21. Computed head declines in the upper aquifer by 1980.



EXPLANATION

——10—— Line of approximate equal head decline
Interval 10 feet (3 metres)

Figure 22. Computed head declines in the upper aquifer by 1990.

Computed reductions in streamflow from 1970 to 1980 and 1990 above selected points on the Yahara River are summarized in table 3. These values appear reasonable indicating that streams and lakes can be treated as constant-head boundaries for the period of the analysis.

The 1980 and 1990 reductions in mean annual flow of the Yahara River at the stream-gaging station were computed as 67 and 81 ft^3/s (1.9 and 2.3 m^3/s), respectively (table 4). The mean annual flow past the stream-gaging station before pumping began was estimated to have been 152 ft^3/s (4.3 m^3/s). Thus, by 1990 a reduction in mean flow of approximately 53 percent can be expected at the gage. This compares to an approximate 34 percent flow reduction in 1970.

The effects of pumping from the upper Yahara River basin will continue to spread into adjacent basins. The model computed that water diverted from these basins to wells in the upper Yahara River basin will increase from 5 ft^3/s to 12 ft^3/s (0.1-0.3 m^3/s) by 1990 (table 4).

The estimated 1980 and 1990 pumpages represent only one of many possible courses that development of the sandstone aquifer may take in future years. These estimates represent, however, one of the more likely courses to be followed if current development trends continue.

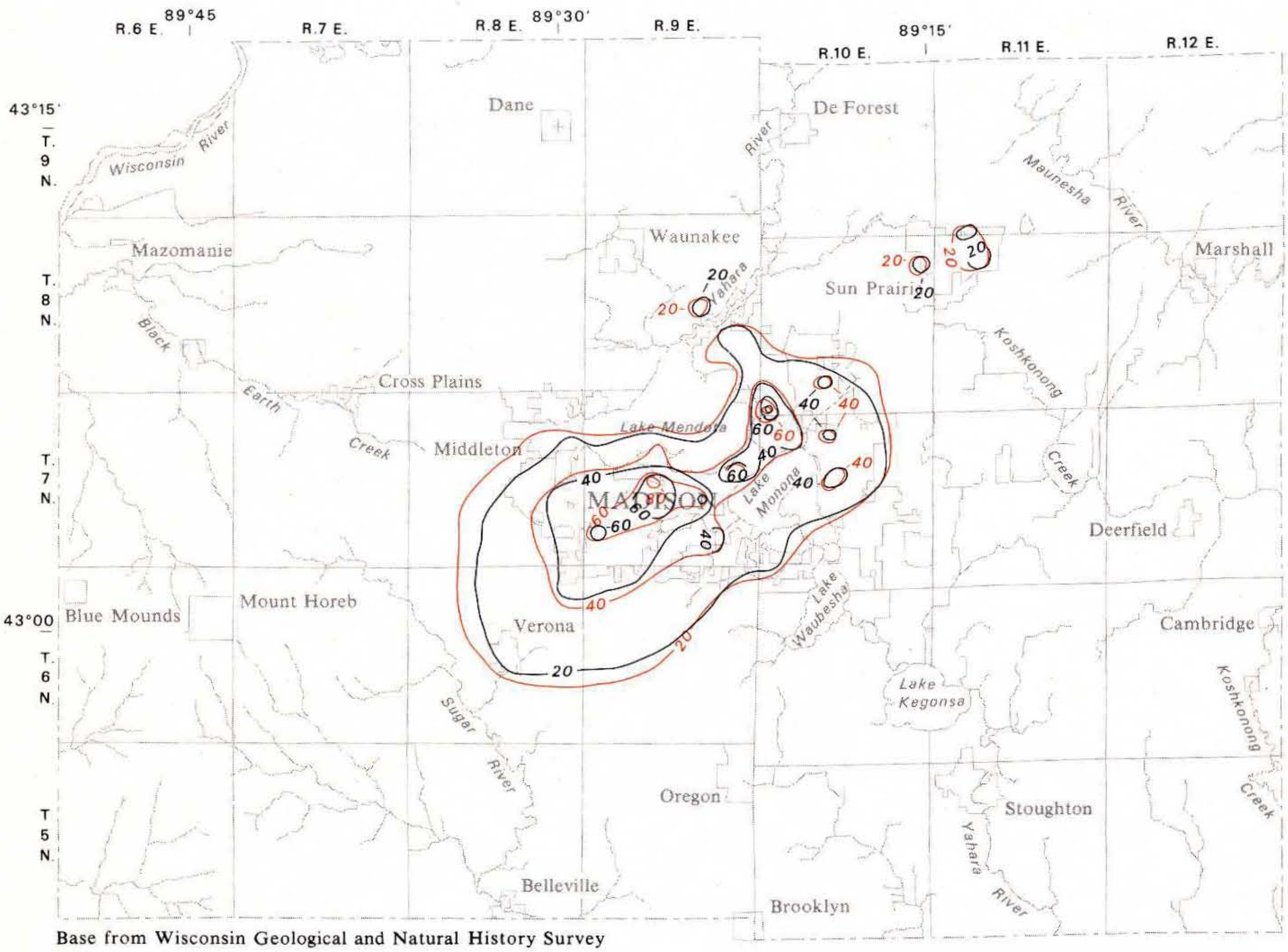
Table 3.--Reductions in streamflow from 1970 to 1980 and 1990 above selected points on the upper Yahara River.

Year	Yahara River above Lake Mendota (ft^3/s)	Outlet from Lake Mendota (ft^3/s)	Outlet from Lake Monona (ft^3/s)	Outlet from Lake Waubesa (ft^3/s)
1980	2	8	12	15
1990	3	13	23	29

Comparisons with Earlier Model

An earlier model for computing head declines in the sandstone aquifer (McLeod, 1975) assumed that head declines in the upper aquifer had a negligible effect on the computations. This assumption resulted in slightly smaller computed head declines in the sandstone aquifer than were computed here (figs. 23 and 24).

The percentage difference between head declines computed by the two models is greatest for small values; it decreases for increased values. Head declines computed by the sandstone aquifer model are generally 10 to 20 percent smaller than those in this report along the 20-ft (6-m) line of equal head decline. They are only 3 to 5 percent smaller at points of maximum head declines.



Base from Wisconsin Geological and Natural History Survey

0 2 4 6 MILES

0 2 4 6 KILOMETRES

EXPLANATION

——20—— Line of approximate equal head decline
McLeod, 1975

——20—— Line of approximate equal head decline
This report

Interval 20 feet (6 metres)

Figure 23. Comparison of computed head declines for the sandstone aquifer, 1980.

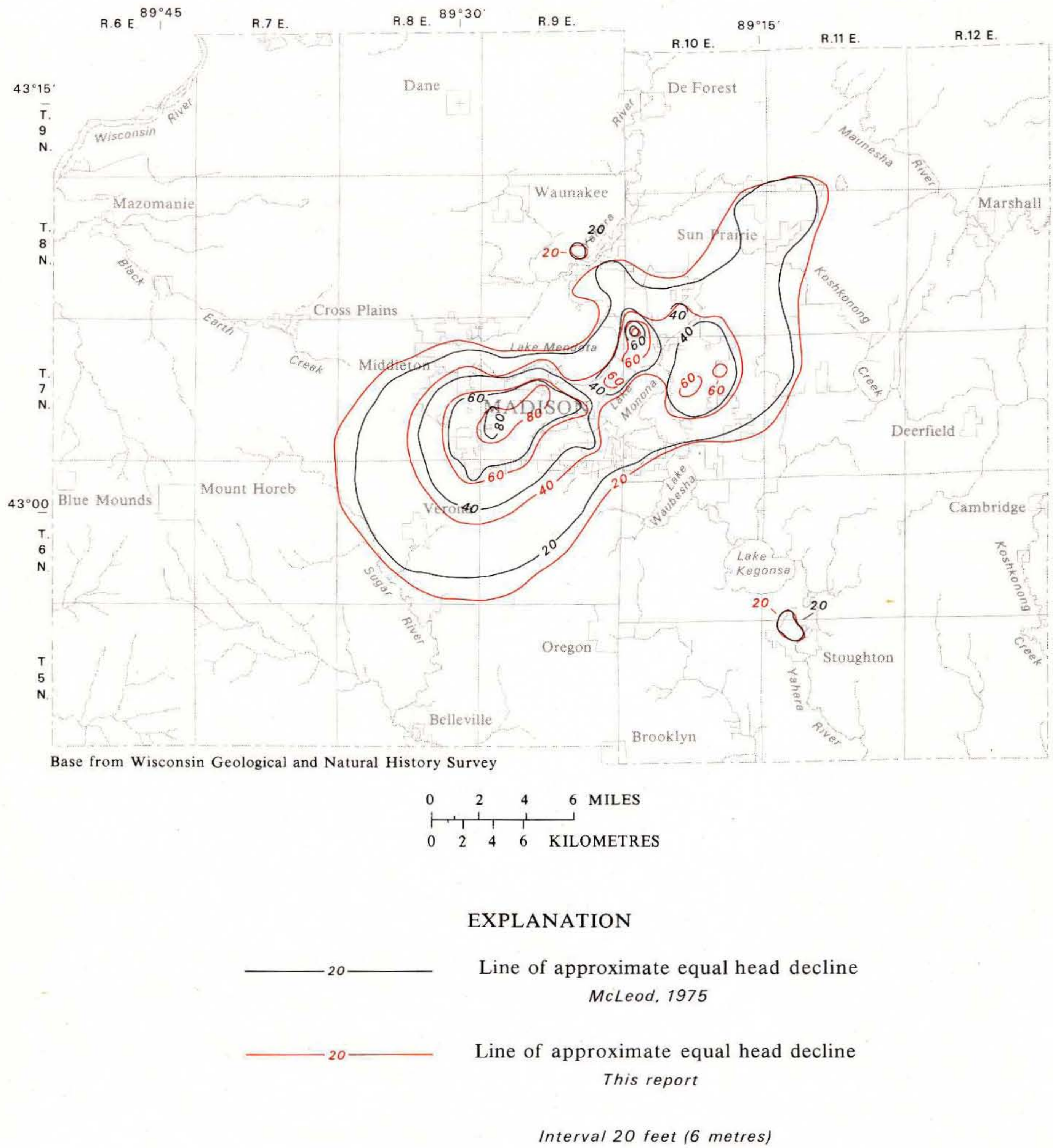


Figure 24. Comparison of computed head declines for the sandstone aquifer, 1990.

Table 4.--Ground-water budget for flow changes in upper Yahara River basin for 1970, 1980, and 1990.

Year	Water diverted to wells		Pumpage from sandstone aquifer within basin	Total pumpage from sandstone aquifer
	From upper Yahara River basin	From adjacent river basins		
1970	52 ft^3/s (91 percent)	5 ft^3/s (9 percent)	57 ft^3/s (37 Mgal/d)	62 ft^3/s (40 Mgal/d)
1980	67 ft^3/s (89 percent)	8 ft^3/s (11 percent)	75 ft^3/s (48 Mgal/d)	82 ft^3/s (53 Mgal/d)
1990	81 ft^3/s (87 percent)	12 ft^3/s (13 percent)	93 ft^3/s (60 Mgal/d)	103 ft^3/s (66 Mgal/d)

SUMMARY AND CONCLUSIONS

A digital-computer program was developed to solve quasi three-dimensional ground-water flow problems. The program uses the iterative alternating direction, implicit technique for solving finite-difference approximations to the partial-differential equations governing flow in the two-layer aquifer system. The program comuptes head changes in both the confined and unconfined aquifers. It also computes the rate and volume of water withdrawn from constant-head boundaries.

The program was used to model the aquifer system in Dane County, Wis., as a confined sandstone aquifer overlain by a leaky upper aquifer and underlain by impermeable bedrock. The physical properties of the aquifer system needed for the model were approximated using aquifer test data and by matching measured 1970 head changes in the system and reductions in streamflow with those computed by the model.

The model was used to compute 1980 and 1990 head declines in the upper and sandstone aquifers and reductions in streamflow caused by pumping the sandstone aquifer. The 1980 and 1990 pumping rates were estimated by an arithmetic projection of the past pumping trend of each ground-water user to 1990. These pumping rates were distributed in the model according to the likely course that aquifer development would take if current development trends continue.

Model results indicated that the aquifer system should be able to supply the water needs of the county well beyond 1990. Maximum head declines in the upper aquifer between 1970 and 1990 are not expected to exceed about 10 ft (3 m). Maximum head declines in the sandstone aquifer between 1970 and 1990 should not exceed about 40 ft (12 m). These declines would not be a serious detriment to the available ground-water supply.

Reductions in flow to streams can be expected to increase along with increased pumpage from the sandstone aquifer. Between 1970 and 1990 increased pumpage would reduce streamflow past the upper Yahara River gaging station by an additional 29 ft^3/s (0.8 m^3/s). Also during this period an additional 7 ft^3/s (0.2 m^3/s) of water that normally would have discharged to adjacent basins would be diverted to wells in the upper Yahara River basin.

Maximum differences between results presented in the sandstone aquifer report (McLeod, 1975) and those presented in this report for computed 1980 and 1990 head declines in the sandstone aquifer occurred in areas where head declines were least. Computed head declines presented in the sandstone aquifer report were generally 10 to 20 percent greater than those in this report in the area of the 20-ft (6-m) line of equal head decline. The difference between results generally decreased for increased head declines. It was only about 3 percent at points of maximum head declines for the 1990 computations.

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